$B_d(\bar{B}_d) \to \rho^{\pm}\pi^{\mp}, \rho^+\rho^-, \pi^+\pi^-$: hunting for alpha

a seminar in memory of

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19 december 2005

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M.V., G.G. Ovanesyan, JETP Letters 81 (2005)361

hep-ph/0506322 (accepted in Yad.Fiz.)

ITEP TH

CKM matrix

$$\overline{(uct)_L}\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

Wolfenstein parametrization: λ, A, ρ, η
expanding in powers of λ:

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda - iA^2\lambda^5\bar{\eta} & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 - iA\lambda^4\bar{\eta} & 1 \end{pmatrix} ,$$

where

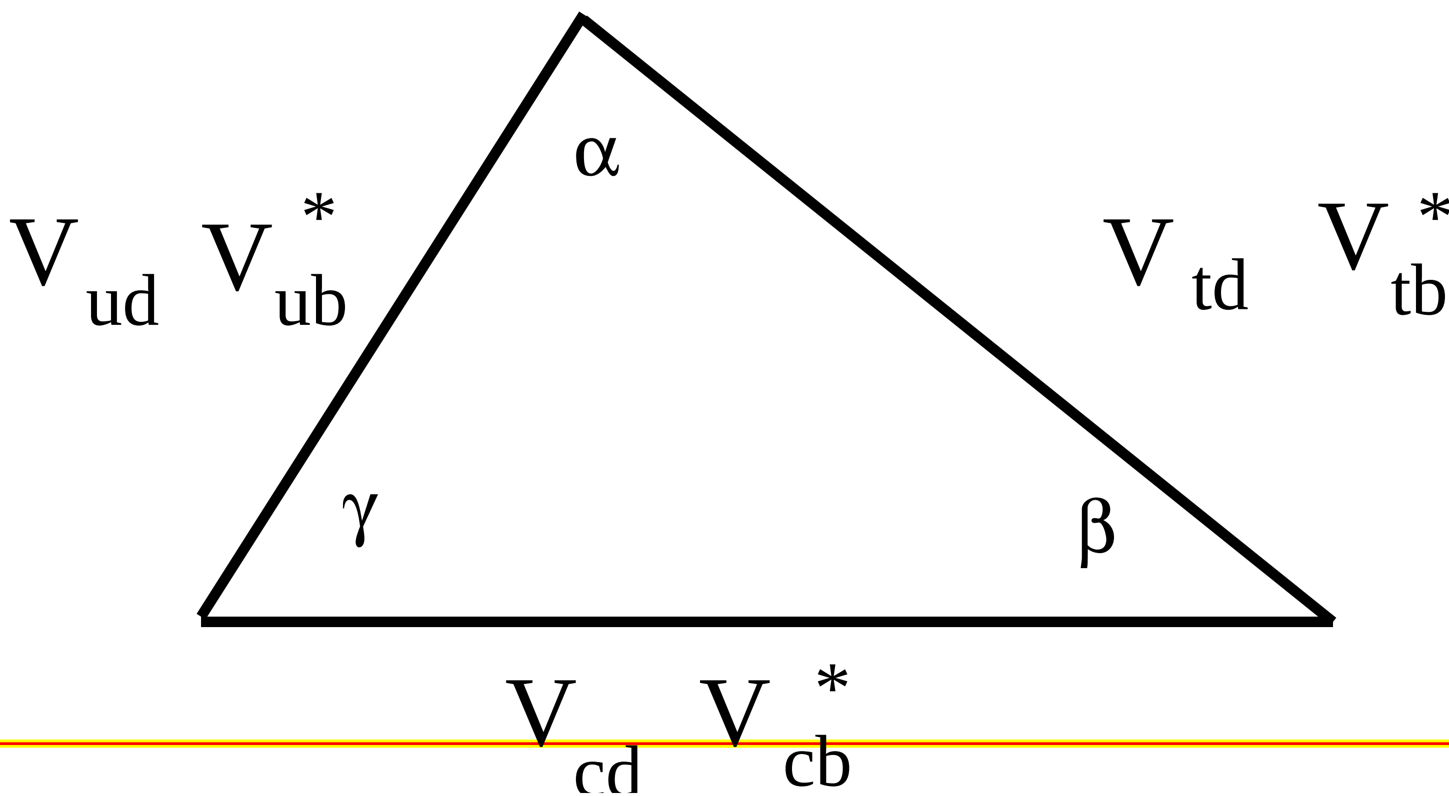
$$\bar{\rho} \equiv \rho(1-\frac{\lambda^2}{2}) \ , \ \bar{\eta} \equiv \eta(1-\frac{\lambda^2}{2})$$

Unitarity triangle

$K \to \pi l \nu : \lambda = 0.226 \pm 0.002 \quad b \to c l \nu : A = 0.82 \pm 0.03$

2 parameters remain: ρ and η. Unitarity triangle:

$$\underset{\sim \lambda^3}{V_{ud}^* V_{ub}} + \underset{\sim \lambda^3}{V_{cd}^* V_{cb}} + \underset{\sim \lambda^3}{V_{td}^* V_{tb}} = 0 \ ,$$



α from $B \to \rho\pi$

$$\frac{dN(B_d(\bar{B}_d) \to \rho^{\pm}\pi^{\mp})}{dt} = (1 \pm A_{CP}^{\rho\pi})e^{-t/\tau}[1 - q(C_{\rho\pi} \pm \Delta C_{\rho\pi})\times$$

$$\times \cos(\Delta m t) + q(S_{\rho\pi} \pm \Delta S_{\rho\pi}) \sin(\Delta m t)]$$

where $q = -1$ describes the case when at $t = 0$ B_d was produced, while $q = 1$ corresponds to $\bar{B}_d$ production at $t = 0$

$$C_{\rho\pi} \pm \Delta C_{\rho\pi} = \frac{1-|\lambda^{\pm\mp}|^2}{1+|\lambda^{\pm\mp}|^2} \quad ; \quad \lambda^{+-} \equiv \frac{q}{p}\frac{\bar{M}^{+-}}{M^{+-}}$$

where parameters q and p enter the expressions for ($B_d, \bar{B}_d$) eigenstates, $q/p = e^{-2i\beta}$

$$A_{CP}^{\rho\pi} = \frac{|M^{+-}|^2 - |\bar{M}^{-+}|^2 + |\bar{M}^{+-}|^2 - |M^{-+}|^2}{|M^{+-}|^2 + |\bar{M}^{-+}|^2 + |\bar{M}^{+-}|^2 + |M^{-+}|^2} \quad ,$$

$$S_{\rho\pi} \pm \Delta S_{\rho\pi} = \frac{2\mathrm{Im}\lambda^{\pm\mp}}{1+|\lambda^{\pm\mp}|^2}$$

$C_{\rho\pi}, \Delta C_{\rho\pi}, A_{CP}^{\rho\pi}, S_{\rho\pi}, \Delta S_{\rho\pi}$ are measured by Belle and

BABAR. Theory: $M, \bar{M}$ - the value of α

α from $B \to \pi^+\pi^-, \rho^+\rho^-$

$$\frac{dN(B_d(\bar{B}_d) \to \pi^+\pi^-)}{dt} = e^{-t/\tau}[1 - q(C_{\pi\pi} \times \cos(\Delta m t) +$$

$$+qS_{\pi\pi}\sin(\Delta m t)]$$

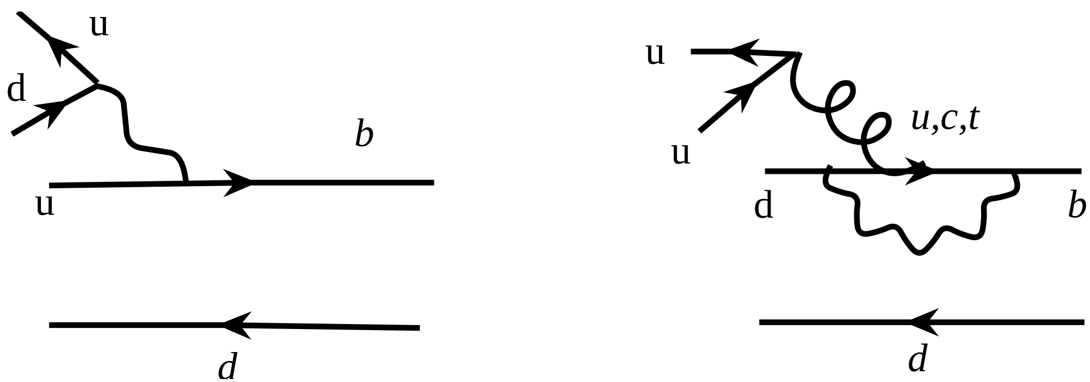
$C_{\pi\pi} = \frac{1-|\lambda_{\pi\pi}|^2}{1+|\lambda_{\pi\pi}|^2}$, $S_{\pi\pi} = \frac{2\mathrm{Im}\lambda_{\pi\pi}}{1+|\lambda_{\pi\pi}|^2}$, $\lambda_{\pi\pi} \equiv \frac{q}{p}\frac{\bar{M}_{\pi^+\pi^-}}{M_{\pi^+\pi^-}}$

Analogous formulas hold for decays to $\rho_L^+\rho_L^-$ (the longitudinal polarization of ρ dominates, $f_L = 0.98 \pm 0.01 \pm 0.02$)

$C_{\pi\pi}, S_{\pi\pi}, C_{\rho\rho}, S_{\rho\rho}$ are measured by Belle and BABAR.

All phenomenology is presented.
How to calculate M?

$b \to u d \bar{u}$



$B \to \pi\pi,\ B \to \rho\rho,\ B \to \rho\pi$ decays

How large is penguin?

G.G. Ovanesyan, M.V., JETP Letters 81 (2005) 449

$P/T = 0$; values of α from all 3 decays agree with each other and with global CKM fit result.

Present paper: $P/T \neq 0$.

effective Hamiltonian

$$\hat{H} = \frac{G_F}{\sqrt{2}} \left[V_{ub} V_{ud}^* (c_1 O_1 + c_2 O_2) - \right.$$

$$\left. - V_{tb} V_{td}^* (c_3 O_3 + c_4 O_4 + c_5 O_5 + c_6 O_6) \right]$$

$$\begin{aligned}
O_1 &= \bar{u}\gamma_\alpha(1+\gamma_5)b\bar{d}\gamma_\alpha(1+\gamma_5)u \\
O_2 &= \bar{d}\gamma_\alpha(1+\gamma_5)b\bar{u}\gamma_\alpha(1+\gamma_5)u \\
O_3 &= \bar{d}\gamma_\alpha(1+\gamma_5)b[\bar{u}\gamma_\alpha(1+\gamma_5)u + \bar{d}\gamma_\alpha(1+\gamma_5)d] \\
O_4 &= \bar{d}_a\gamma_\alpha(1+\gamma_5)b^c[\bar{u}_c\gamma_\alpha(1+\gamma_5)u^a + \bar{d}_c\gamma_\alpha(1+\gamma_5)d^a] \\
O_5 &= \bar{d}\gamma_\alpha(1+\gamma_5)b[\bar{u}\gamma_\alpha(1-\gamma_5)u + \bar{d}\gamma_\alpha(1-\gamma_5)d] \\
O_6 &= \bar{d}_a\gamma_\alpha(1+\gamma_5)b^c[\bar{u}_c\gamma_\alpha(1-\gamma_5)u^a + \bar{d}_c\gamma_\alpha(1-\gamma_5)d^a]
\end{aligned}$$

$$c_1 = 1.1, c_2 = -0.3, c_3 = 0.02, c_4 = -0.04, c_5 = 0.01, c_6 = -0.04$$

$c_3 - c_6$ are very small; penguins in charmless strangeless B decays are less important than in kaon decays

factorization

In order to calculate matrix elements of $\hat{H}$ we suppose that factorization occurs:
$< M_1 M_2 | j_1 j_2 | B > = < M_1 | j_1 | B >< M_2 | j_2 | 0 >$,
which allows to transform $\hat{H}$ in the following way:

$$\hat{H} = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \{ a_1 \bar{u} \gamma_\alpha (1+\gamma_5) b \bar{d} \gamma_\alpha (1+\gamma_5) u -$$

$$- \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} [a_4 \bar{u} \gamma_\alpha (1+\gamma_5) b \bar{d} \gamma_\alpha (1+\gamma_5) u -$$

$$- 2 a_6 \bar{u} (1+\gamma_5) b \bar{d} (1-\gamma_5) u] \}$$

where $a_1 = c_1 + \frac{1}{3} c_2 = 1.04$, $a_4 = c_4 + \frac{1}{3} c_3 = -0.031$, $a_6 = c_6 + \frac{1}{3} c_5 = -0.042$

$B \to \pi^+\pi^-$

Up to a common factor which includes constant f_π and $B \to \pi$ transition formfactor we get:

$$\frac{M(\bar{B}_d \to \pi^+\pi^-)}{V_{ub}V_{ud}^*} \sim a_1 - \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}\left[a_4 + \frac{2m_\pi^2}{(m_u+m_d)(m_b-m_u)}a_6\right]e^{i\delta}$$

Aleksan, Buccella,... (1995) BUT $\delta = 0$

$\delta \neq 0$ to get nonzero direct CP asymmetries.

With the help of the relation

$$\frac{V_{td}^* V_{tb}}{V_{ud}^* V_{ub}} = e^{i(\pi-\alpha)}\frac{\sin\gamma}{\sin\beta}$$

we finally obtain:

$M(\bar{B}_d \to \pi^+\pi^-) \sim V_{ub}V_{ud}^*[1 + 0.14e^{i(\pi-\alpha+\delta)}]$, where we put a_1 equal to one.

$B \to \rho^+ \rho^-$

a_6 does not contribute to the decay amplitude since $< \rho|\bar{d}(1-\gamma_5)u|0 >= 0$:

$$\frac{M(\bar{B}_d \to \rho_L^+ \rho_L^-)}{V_{ub}V_{ud}^*} \sim a_1 - \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} a_4 e^{i\delta} \ ,$$

$$M(\bar{B}_d \to \rho_L^+ \rho_L^-) \sim V_{ub}V_{ud}^*[1 + 0.07 e^{i(\pi - \alpha + \delta)}]$$

$B \to \rho\pi$

The amplitude $\bar{M}^{-+}$ corresponds to ρ^- production from vacuum by $(\bar{d}u)$ current, so the term proportional to a_6 does not contribute to it:

$$\frac{\bar{M}^{-+}}{V_{ub}V_{ud}^*} = A\left[a_1 - \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}a_4 e^{i\delta_-}\right]$$

$$\bar{M}^{-+} = AV_{ub}V_{ud}^*\left[1 + 0.07e^{i(\pi-\alpha+\delta_-)}\right]$$

where A is the complex number.

$$\frac{\bar{M}^{+-}}{V_{ub}V_{ud}^*} = B\left\{a_1 - \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}\left[a_4 - \frac{2m_\pi^2}{(m_b+m_u)(m_u+m_d)}a_6\right]e^{i\delta_+}\right\} \approx B$$

$M^{+-} = \bar{M}^{-+}, M^{-+} = \bar{M}^{+-}$ with c.c. CKM matrix elements

α from $B \to \rho\pi$

$\alpha,\ \delta_- \equiv \delta$, $\quad A/B \equiv |A/B|e^{i\tilde{\delta}}$ - 4 parameters; 5 observables

$$\Delta C_{\rho\pi}(Belle, BABAR) = 0.22 \pm 0.10$$

$$|A/B|^2 = \frac{1 + \Delta C_{\rho\pi}}{1 - \Delta C_{\rho\pi}} = 1.56 \pm 0.33$$

$$C_{\rho\pi} = 0.31 \pm 0.10 \ : \quad \sin\delta = \frac{(1 + |A/B|^2)^2}{0.28|A/B|^2} C_{\rho\pi} = 4.6 \pm 1.5$$

$$A_{CP}^{\rho\pi} = -0.102 \pm 0.045 \ : \quad \sin\delta = \frac{1 + |A/B|^2}{0.14|A/B|^2} A_{CP}^{\rho\pi} = -1.2 \pm 0.5$$

3.5σ deviation - largest we encounter; average:
$\sin\delta = -0.62 \pm 0.47$

$\sin\tilde{\delta} = \Delta S_{\rho\pi} = 0.09 \pm 0.13 \ : \quad \tilde{\delta} \approx 0,$ or $\quad \tilde{\delta} \approx \pi$

$$\sin 2\alpha = S_{\rho\pi}/\cos\tilde{\delta} + 0.07\cos\delta \ , \quad S_{\rho\pi} = -0.13 \pm 0.13$$

$|\cos\tilde{\delta}| = 1 \ , \quad |\cos\delta| = 0.88 \pm 0.12$ (or unknown)

$\tilde{\delta} \approx 0 \ , \ \ \delta \approx 0 : \quad \alpha = 92^o \pm 4^o$

$\tilde{\delta} \approx 0 \ , \ \ \delta \approx \pi : \quad \alpha = 96^o \pm 4^o$

(or $\alpha = 94^o \pm 4^o \pm 2^o$; uncertainty from poor knowledge of δ is small since P/T is small)

$\tilde{\delta} \approx \pi \ , \ \ \delta \approx 0 : \quad \alpha = 84^o \pm 4^o$

$\tilde{\delta} \approx \pi \ , \ \ \delta \approx \pi : \quad \alpha = 88^o \pm 4^o$

independently we "know" that δ is small, so 2 possibilities remain: $92^o \pm 4^o$ or $84^o \pm 4^o$

α from $B \to \rho^+\rho^-$

$C_{\rho\rho} = 0.14 \sin\alpha \sin\delta \approx 0.14 \sin\delta = -0.02 \pm 0.17$
(BABAR and Belle averaged)
$\sin\delta = -0.2 \pm 1.4\ ;\ |\cos\delta| = 0.88 \pm 0.12$

$S_{\rho\rho} = \sin 2\alpha - 0.14 \cos\delta = -0.21 \pm 0.22$
$\delta \approx 0 :\ \alpha = 92^o \pm 7^o$
($\delta \approx \pi :\ \alpha = 100^o \pm 7^o$)

BABAR isospin analysis : $\alpha = 100^o \pm 13^o$
smallness of Br ($B \to \rho^o\rho^0$)

α from $B \to \pi^+\pi^-$

$C_{\pi\pi} = 0.28 \sin\alpha \sin\delta \approx 0.28 \sin\delta = -0.37 \pm 0.10$
($C_{\pi\pi}^{BABAR} = -0.09 \pm 0.15$, penguin is small;
$C_{\pi\pi}^{Belle} = -0.56 \pm 0.13$, penguin is big)
$\sin\delta = -1.32 \pm 0.35\ , \quad |\cos\delta| = 0.55 \pm 0.25$

$S_{\pi\pi} = \sin 2\alpha - 0.28 \cos\delta = -0.50 \pm 0.12$

$\delta \approx 0 : \ \alpha = 100^o \pm 4^o$
($\delta \approx \pi : \ \alpha = 110^o \pm 4^o$)

$B_d, B_u \to \pi\pi$ puzzle

Experiment:

$$\tfrac{1}{2}\left[Br(B_d \to \pi^+\pi^-) + Br(\bar{B}_d \to \pi^+\pi^-)\right] = 5.0 \pm 0.4$$
$$\tfrac{1}{2}\left[Br(B_d \to \pi^0\pi^0) + Br(\bar{B}_d \to \pi^0\pi^0)\right] = 1.45 \pm 0.29 \qquad \times 10^{-6}$$
$$Br(B_u \to \pi^+\pi^0) = 5.5 \pm 0.6$$

Theory (factorization): $\Gamma_{\pi^0\pi^0}/\Gamma_{\pi^+\pi^-} \sim 10^{-2}$; factor 30.

Way out: $M_\pm = A_2 e^{i\delta_t} + A_0$,

....

$\delta_t = 53^o \pm 7^o$, while for A_2 and A_0 factorization works very well.

$$\begin{aligned}\delta\alpha &= -\left|\frac{V_{td}}{V_{ub}}\right| P \frac{1+C_{+-}}{2\sqrt{3}} \sin\alpha \times \\ &\times \frac{\sqrt{2}A_0\cos\delta_p + A_2\cos(\delta_p - \delta_t)}{1/12A_2^2 + 1/6A_0^2 + 1/(3\sqrt{2})A_0A_2\cos\delta_t} \approx \\ &\approx -2.60(1+C_{+-})P\cos(\delta_p - \chi) \approx \\ &\approx -0.14\cos(\delta_p - \chi)\end{aligned}$$

$$\chi = \arcsin\frac{A_2\sin\delta_t}{\sqrt{2A_0^2 + A_2^2 + 2\sqrt{2}A_0A_2\cos\delta_t}} \approx 23^o$$

Numerically shift of α is small.

Conclusions

$\delta \approx 0: \ \alpha_{\rho\rho,\pi\pi} = 99^o \pm 4^o$

taking into account $\rho\pi$ data we finally obtain:

$\alpha_{b\to u\bar{u}d} = 95^o \pm 3^o$ if $\tilde{\delta} \approx 0$

$\alpha_{b\to u\bar{u}d} = 91^o \pm 3^o$ if $\tilde{\delta} \approx \pi$

Global CKM fit results :

$\alpha_{\rm UTfit} = 94^o \pm 8^o \ , \ \ \alpha_{\rm CKMfitter} = 94 \pm 10^o$

Thus the accuracy of the present day knowledge of α can be close to that of β: $\beta = 22^o \pm 2^o$