Results of the region-exchange based analysis of the reaction $\pi^- p \to \pi\pi n$

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Two body reactions:

Reaction	Experiment	Reaction	Experiment
$\pi^+\pi^- \to \pi^+\pi^-$ (all waves)	CERN-München		
$\pi\pi \to \pi^0\pi^0$ (S-wave)	GAMS	$\pi\pi \to \pi^0\pi^0$ (S-wave)	E852
$\pi\pi \to \eta\eta$ (S-wave)	GAMS	$\pi\pi \to \eta\eta'$ (S-wave)	GAMS
$\pi\pi \to K\bar{K}$ (S-wave)	BNL	$K^-\pi^+ \to K^-\pi^+$ (S-wave)	LASS

Three body reactions from Crystal Barrel: (L-liquid, G-gaseous targets).

Reaction	Target	Reaction	Target	Reaction	Target
$\bar{p}p \to \pi^0\pi^0\pi^0$	(L) H_2	$\bar{p}p \to \pi^+\pi^0\pi^-$	(L) H_2	$\bar{p}p \to K_S K_S \pi^0$	(L) H_2
$\bar{p}p \to \pi^0\eta\eta$	(L) H_2	$\bar{p}n \to \pi^0\pi^0\pi^-$	(L) D_2	$\bar{p}p \to K^+K^-\pi^0$	(L) H_2
$\bar{p}p \to \pi^0\pi^0\eta$	(L) H_2	$\bar{p}n \to \pi^-\pi^-\pi^+$	(L) D_2	$\bar{p}p \to K_L K^\pm \pi^\mp$	(L) H_2
$\bar{p}p \to \pi^0\pi^0\pi^0$	(G) H_2			$\bar{p}n \to K_S K_S \pi^-$	(L) D_2
$\bar{p}p \to \pi^0\eta\eta$	(G) H_2			$\bar{p}n \to K_S K^- \pi^0$	(L) D_2
$\bar{p}p \to \pi^0\pi^0\eta$	(G) H_2				

K-matrix for S and D-waves.

In the K-matrix form the unitarity condition is satisfied if:

$$A^J_{m\to n}(s) = \sum_i \hat{K}^J_{mi}(I - i\hat{\rho}^J(s)\hat{K}^J)^{-1}_{in}$$

where $\hat{\rho}$ is the diagonal matrix with elements:

$$\rho^J_{ii}(s) = \frac{2\sqrt{-k^2_{i\perp}}}{\sqrt{s}}(-k^2_{i\perp})^J \, .$$

In the present work we parameterized the elements of the K-matrix as follows:

$$K^J_{mn} = \sum_\alpha \frac{1}{B_J(k^2_{m\perp}, r_\alpha)} \left(\frac{g^{\alpha(J)}_m g^{\alpha(J)}_n}{M^2_\alpha - s} \right) \frac{1}{B_J(k^2_{n\perp}, r_\alpha)} + \frac{f^{(J)}_{mn}}{B_J(k^2_{m\perp}, r_0) B_J(k^2_{n\perp}, r_0)}$$

and P-vector as

$$P^J_m = \sum_\alpha \frac{1}{B_J(k^2_{m\perp}, r_\alpha)} \left(\frac{\Lambda_{\alpha(J)} g^{\alpha(J)}_n}{M^2_\alpha - s} \right) \frac{1}{B_J(k^2_{n\perp}, r_\alpha)} + \frac{F^{(J)}_n}{B_J(k^2_{m\perp}, r_0) B_J(k^2_{n\perp}, r_0)}$$

For the description of the 00^{++} wave in the mass region below 1900 MeV, 5 K-matrix poles are needed:

$$
\begin{aligned}
&f_0^{\textbf{bare}}(680 \pm 100), \quad \psi = (0.45 \pm 0.1)n\bar{n} - (0.89 \pm 0.05)s\bar{s}\ ,\\
&f_0^{\textbf{bare}}(1230 \pm 30), \quad \psi = (0.9^{+0.05}_{-0.2})n\bar{n} + (0.45^{+0.3}_{-0.1})s\bar{s}\ ,\\
&f_0^{\textbf{bare}}(1260 \pm 30), \quad \psi = (0.93^{+0.02}_{-0.1})n\bar{n} + (0.37^{+0.2}_{-0.06})s\bar{s}\ ,\\
&f_0^{\textbf{bare}}(1600 \pm 50), \quad \psi = (0.95 \pm 0.05)n\bar{n} + (0.3^{+014}_{-0.4})s\bar{s}\ ,\\
&f_0^{\textbf{bare}}(1810 \pm 50), \quad \psi = \begin{cases} (0.10 \pm 0.05)n\bar{n} + (0.995^{+0.005}_{-0.015})s\bar{s}, \\ \qquad \big(\textbf{Solution } I\big), \\ (0.67 \pm 0.08)n\bar{n} - (0.74 \pm 0.08)s\bar{s}, \\ \qquad \big(\textbf{Solution } II\big). \end{cases}
\end{aligned}
$$

Experimental data used in the fit do not fix unambiguously the flavor wave function of $f_0^{\textbf{bare}}(1810 \pm 50)$: two solutions are found for it.

The scattering amplitude has five poles in the energy complex plane, four of them correspond to relatively narrow resonances while the fifth resonance is very broad:

$$f_0(980) \to \quad (1015 \pm 15) - i(43 \pm 8) \quad \textbf{MeV},$$

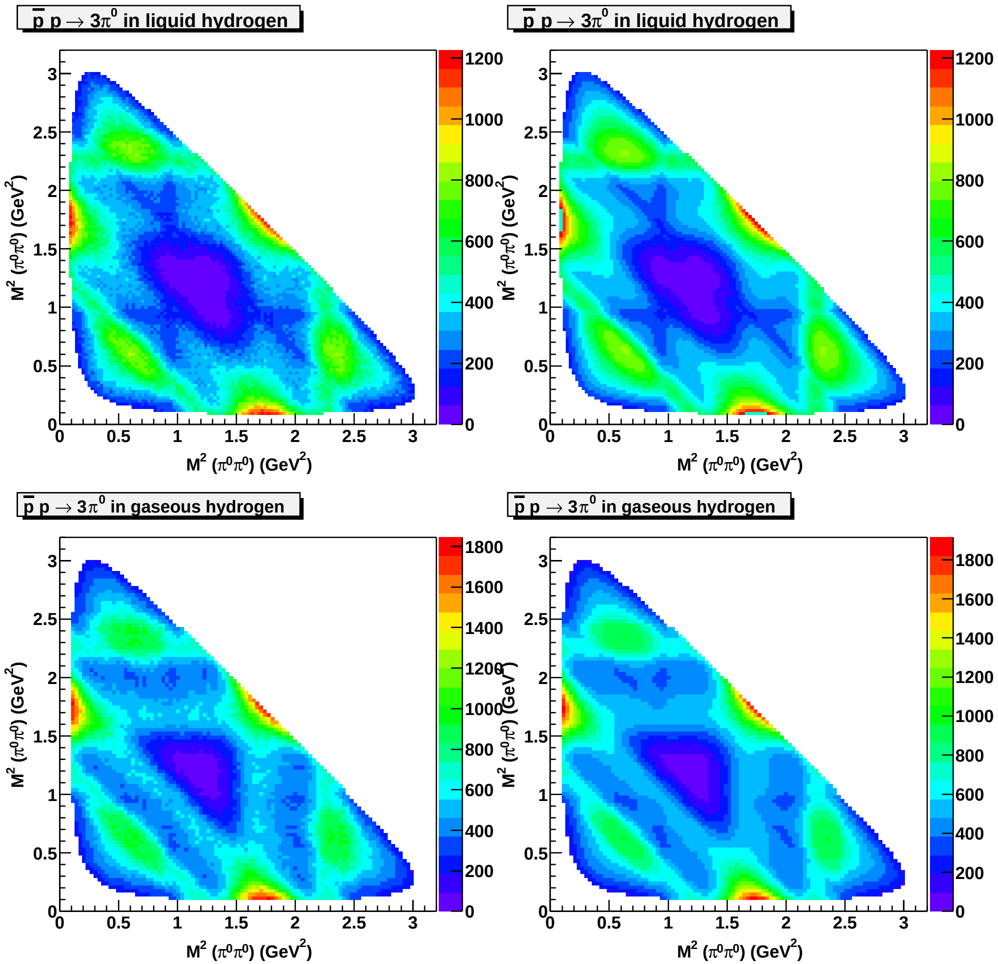
$$f_0(1370) \to \quad (1310 \pm 20) - i(160 \pm 20) \quad \textbf{MeV},$$

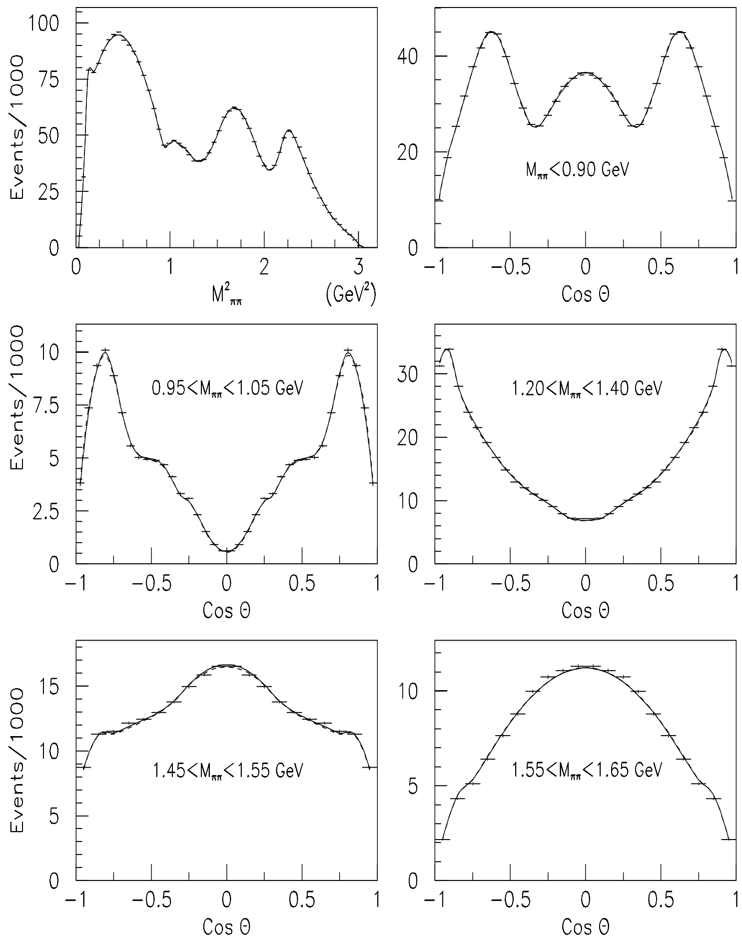
$$f_0(1500) \to \quad (1496 \pm 8) - i(58 \pm 10) \quad \textbf{MeV},$$

$$f_0(1530) \to \quad (1530^{+90}_{-250}) - i(560 \pm 140) \quad \textbf{MeV},$$

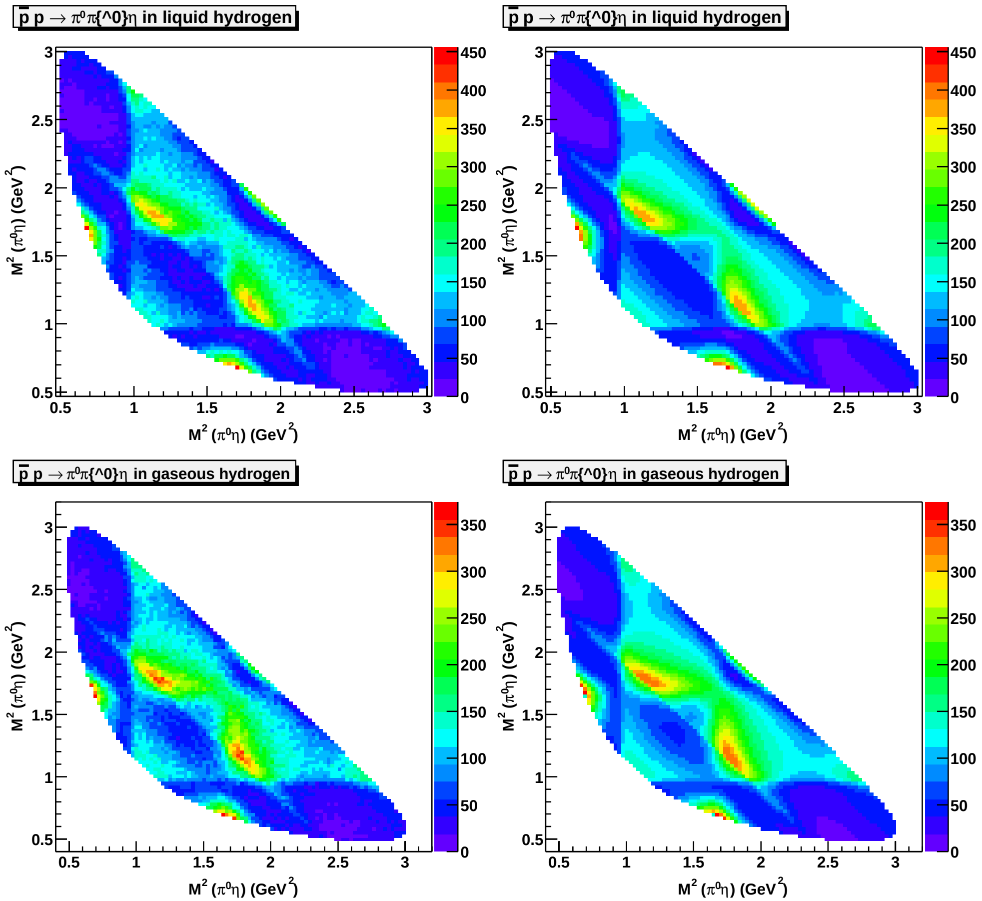
$$f_0(1780) \to \begin{cases} (1780 \pm 30) - i(140 \pm 20) \ \textbf{MeV}, \\ (\textbf{Solution } I), \\ (1780 \pm 50) - i(220 \pm 50) \ \textbf{MeV}, \\ (\textbf{Solution } II). \end{cases}$$

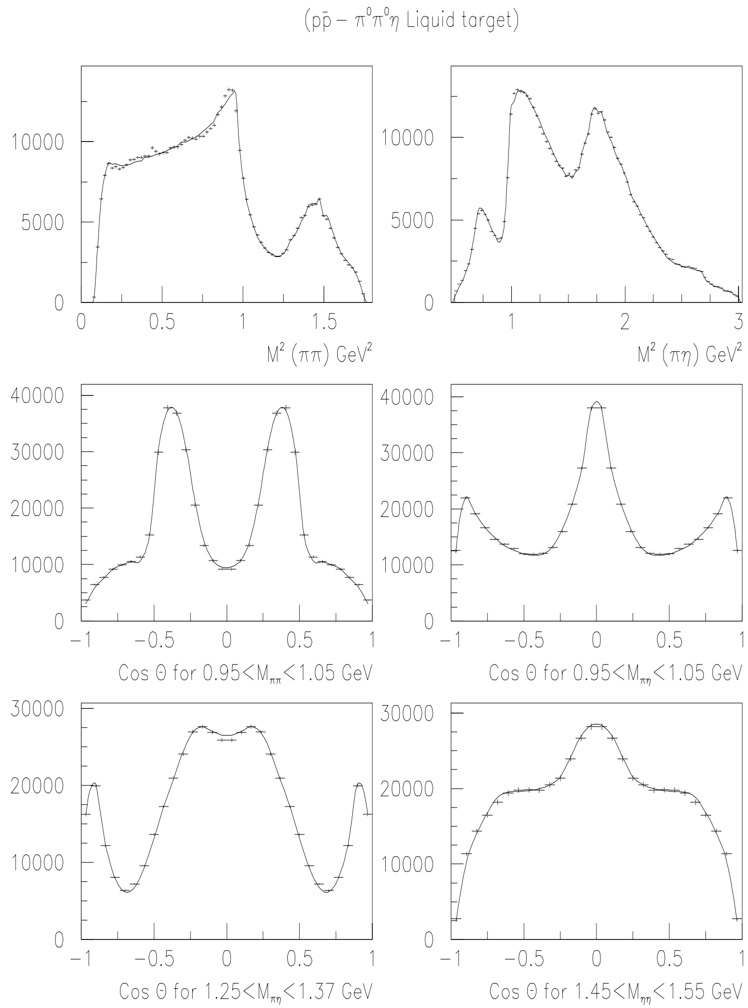
The description of $p\bar{p} \to 3\pi^0$ CB-LEAR data



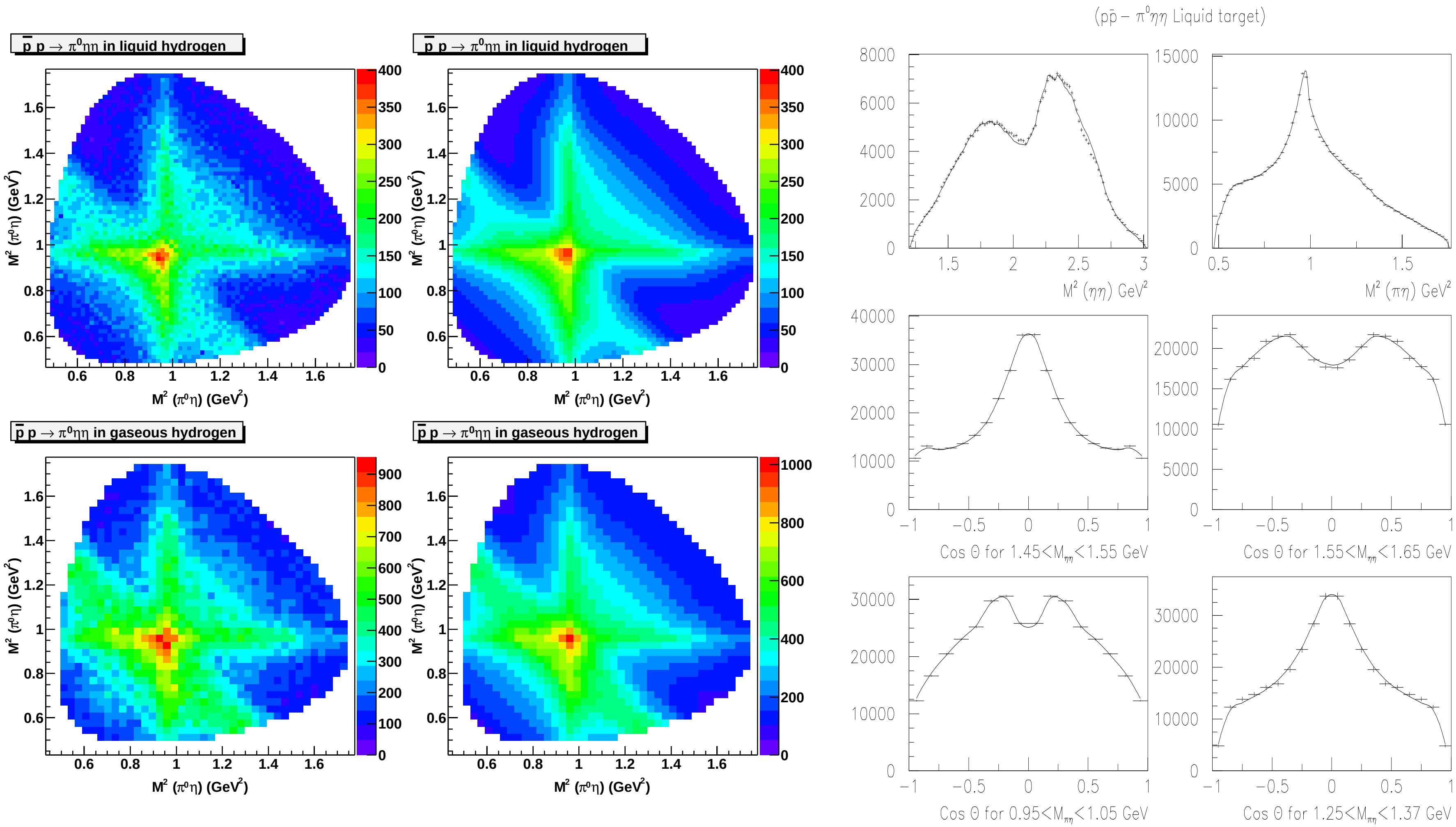


The description of $p\bar{p} \longrightarrow \pi^0\pi^0\eta$ CB-LEAR data





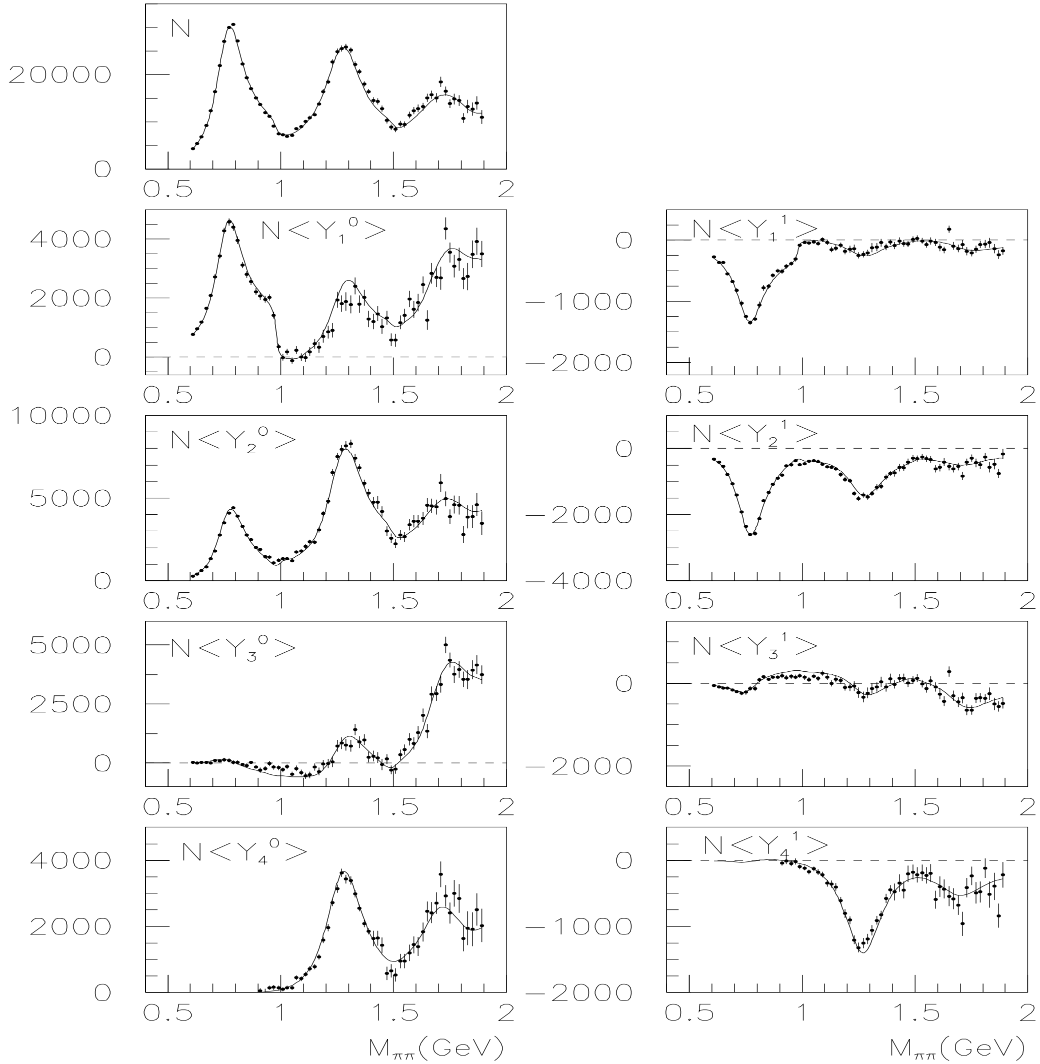
The description of $p\bar{p} \to \pi^0\eta\eta$ CB-LEAR data



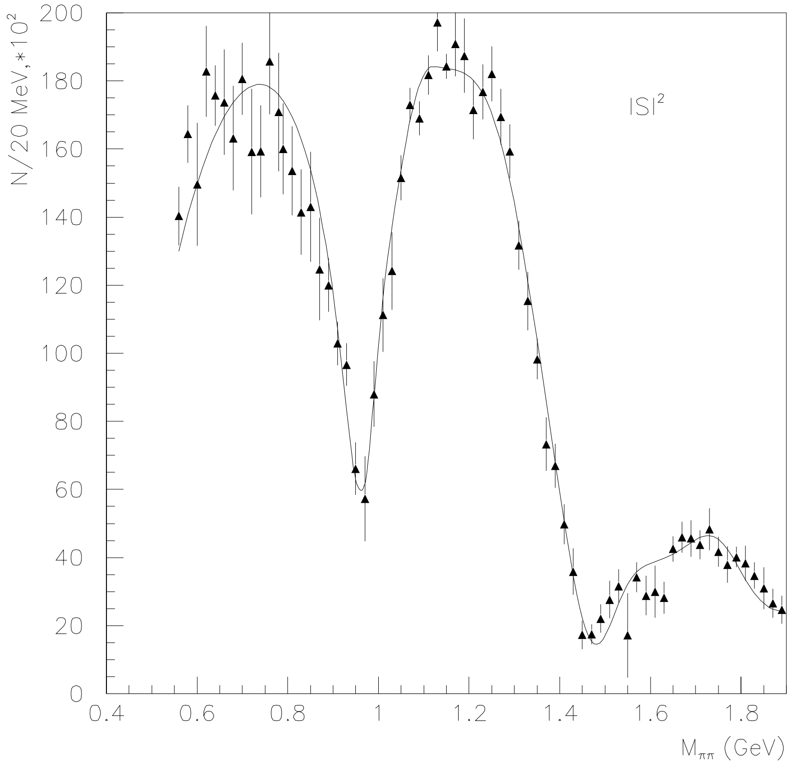
Description of the CERN-Munich data

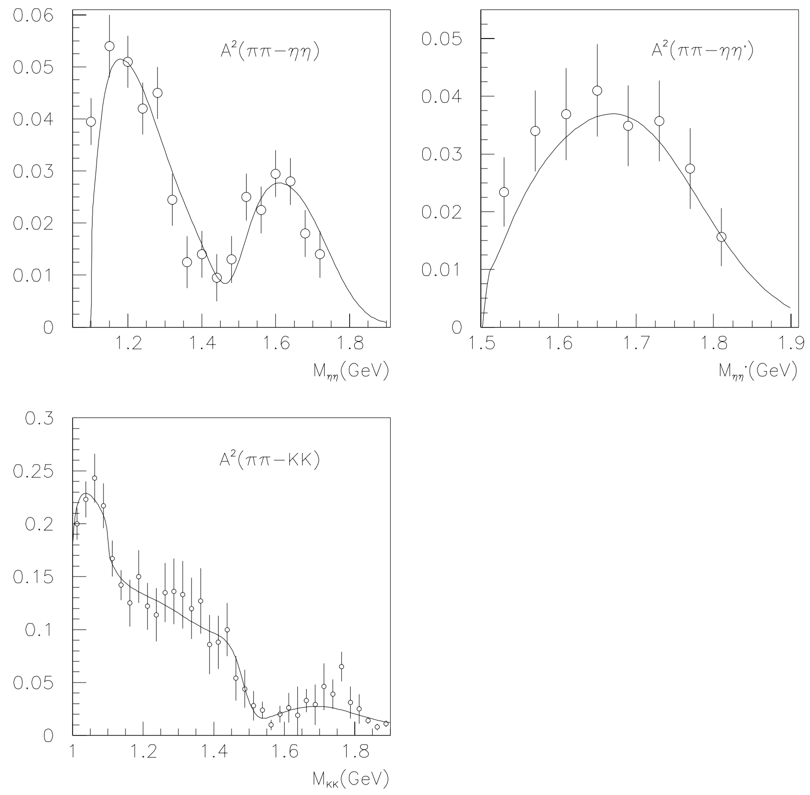
$$\pi^- p \to \pi^+ \pi^- n$$

$$A_{1j} = K_{1m}(I - i\hat{\rho}(s)\hat{K})^{-1}_{mj}$$



The two body scattering data obtained by GAMS play a crucial role for obtaining unambiguous solution.





Problems in the analysis of the $\pi N \to XN$ reactions

The πN reaction with large energy of initial pion should be described by t-exchanges.

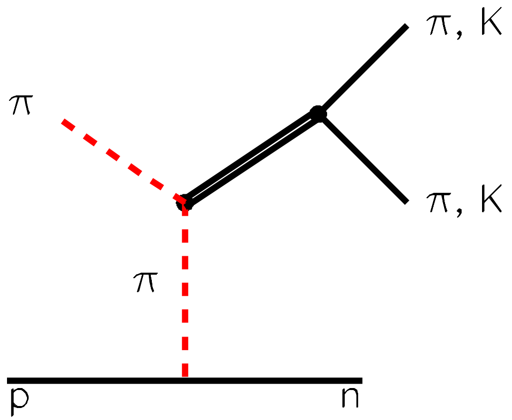
However:

1. **There is no analysis of the data based on the particle exchanges: there are only models.**

2. **There is no a solid analysis which preserves unitarity and includes all known states in P and D-waves. This is should be important at small t where the π exchange is a dominant one and data are close to the unitarity limit.**

3. **Most of the models have problems at large t where exchanges of particles with large spin play a significant role.**

Cross section for the reactions $\pi N \to \pi\pi N, KKN, \eta\eta N$



$$d\sigma = \frac{(2\pi)^4 |A|^2}{8\sqrt{s_{\pi N}}|\vec{p}_2|} d\Phi(p_1 + p_2, k_1, k_2, k_3)$$

$$d\Phi(p_1 + p_2, k_1, k_2, k_3) = (2\pi)^3 d\Phi(P, k_1, k_2)\, d\Phi(p_1 + p_2, P, k_3)\, ds\,,$$

Assuming that amplitude depends only on t and s:

$$d\Phi(p_1 + p_2, P, k_3) \quad = \quad \frac{1}{(2\pi)^5} \frac{dt}{8|\vec{p}_2|\sqrt{s_{\pi N}}} \qquad t = (k_3 - p_2)^2$$

and

$$d\Phi(P, k_1, k_2) = \frac{1}{(2\pi)^5} \rho(s) d\Omega \qquad \rho(s) = \frac{1}{16\pi} \frac{2|\vec{k}_1|}{\sqrt{s}},$$

Then:

$$d\sigma = \frac{(2\pi)^4 |A|^2 (2\pi)^3}{8|\vec{p_2}|\sqrt{s_{\pi N}}} \frac{1}{(2\pi)^5} \frac{dt 2M\, dM\, d\Phi(P, k_1, k_2)}{8|\vec{p_2}|\sqrt{s_{\pi N}}} = \frac{(M|A|^2 \rho) dt\, dM\, d\Omega}{(2\pi)^3 32 |\vec{p_2}|^2\, s_{\pi N}}$$

Unitarity relation:

$$Im A = \rho(s) |A|^2$$

And the cross section can be expressed in the terms of spherical functions:

$$\frac{d^4\sigma}{dt\, dM\, d\Omega} = N \sum_l \left(< Y_l^0 > Y_l^0(\Omega) + \sum_{m=0}^{l} 2 < Y_l^m > Re\, Y_l^m(\Omega) \right)$$

CERN-Munich approach

The CERN-Munich model was developed for the analysis of the data on $\pi^- p \to \pi^+\pi^- n$ reaction and based partly on the absorbtion model but mostly on the phenomenological observations.

$$|A|^2 = |\sum_{J=0} A_J^0 Y_J^0 + \sum_{J=1} A_J^- ReY_J^1|^2 + |\sum_{J=1} A_J^+ ReY_J^1|^2$$

Additional assumptions:

1) helicity 1 amplitudes are equal for natural and unnatural exchanges:

$$A_J^{(-)} = A_J^{(+)}$$

2) The ratio of the $A_J^{(-)}$ and the A_J^0 amplitudes is a polynomial over mass of the two pion system which does not depend on J up to total normalization.

$$A_J^{(-)} = \frac{A_J^0}{C_J \sum\limits_{n=0}^{3} b_n M^n},$$

GAMS, VES and BNL approach

The Cern-Munich approach does not work for large t and does not work for many other final states.

The πN data are decomposed as a sum of amplitudes with angular dependence defined by spherical functions:

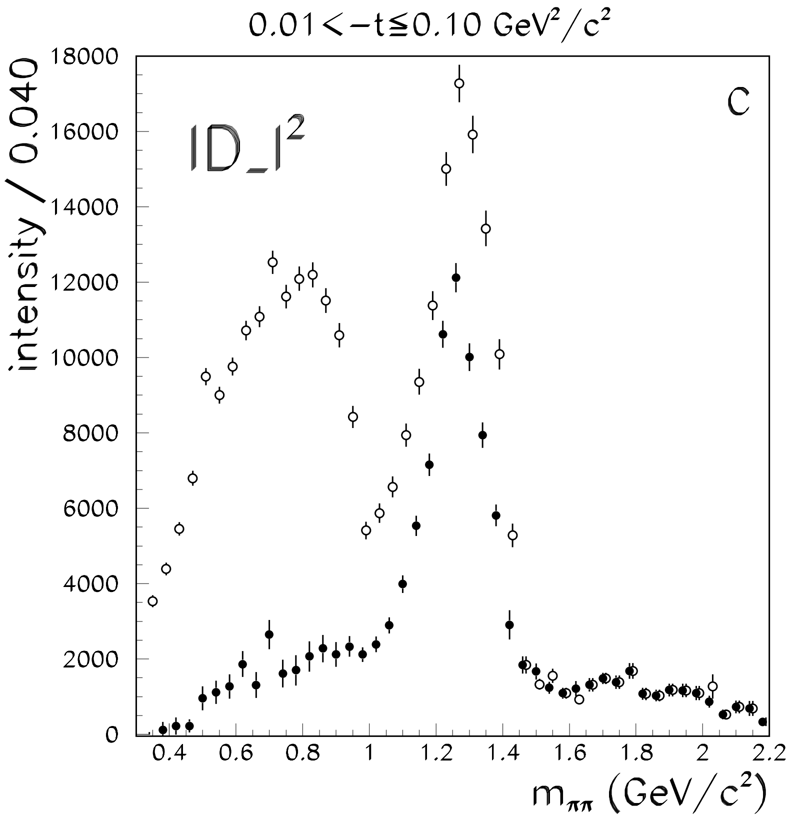
$$|A^2| = |\sum_{J=0} A_J^0 Y_J^0 + \sum_{J=1} A_J^- \sqrt{2}\, Re\, Y_J^1|^2 + |\sum_{J=1} A_J^+ \sqrt{2}\, Im\, Y_J^1|^2$$

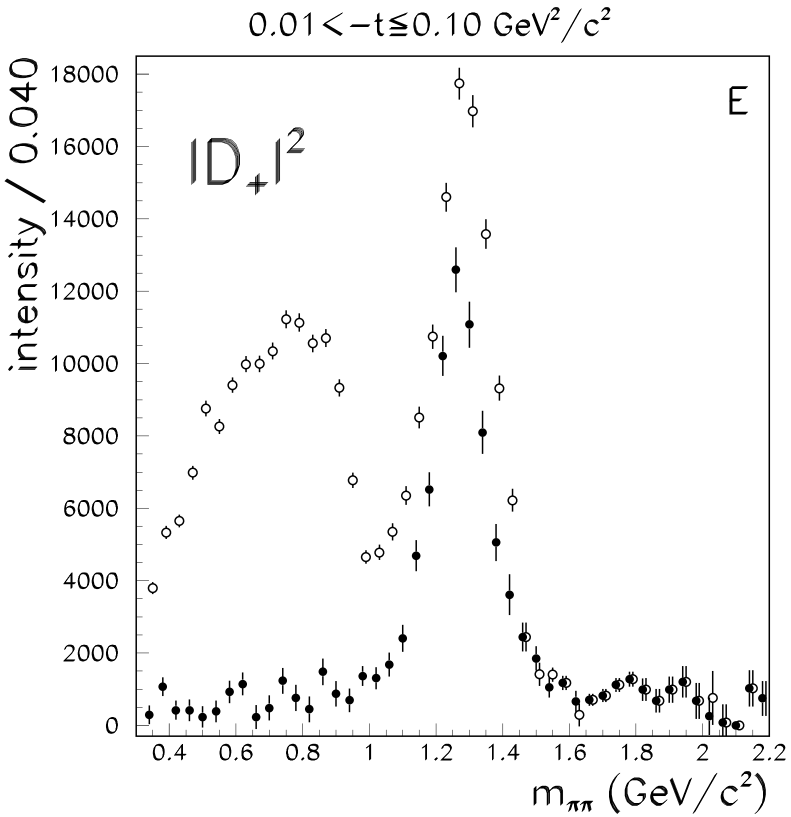
Here the A_J^0 functions are called $S_0, P_0, D_0, F_0 \ldots$, the A_J^- functions defined as $P_-, D_-, F_-, \ldots$ and the A_J^+ functions as $P_+, D_+, F_+, \ldots$.

No assumptions that helicity 1 amplitudes with natural and unnatural exchanges are equal each to another.

BNL analysis

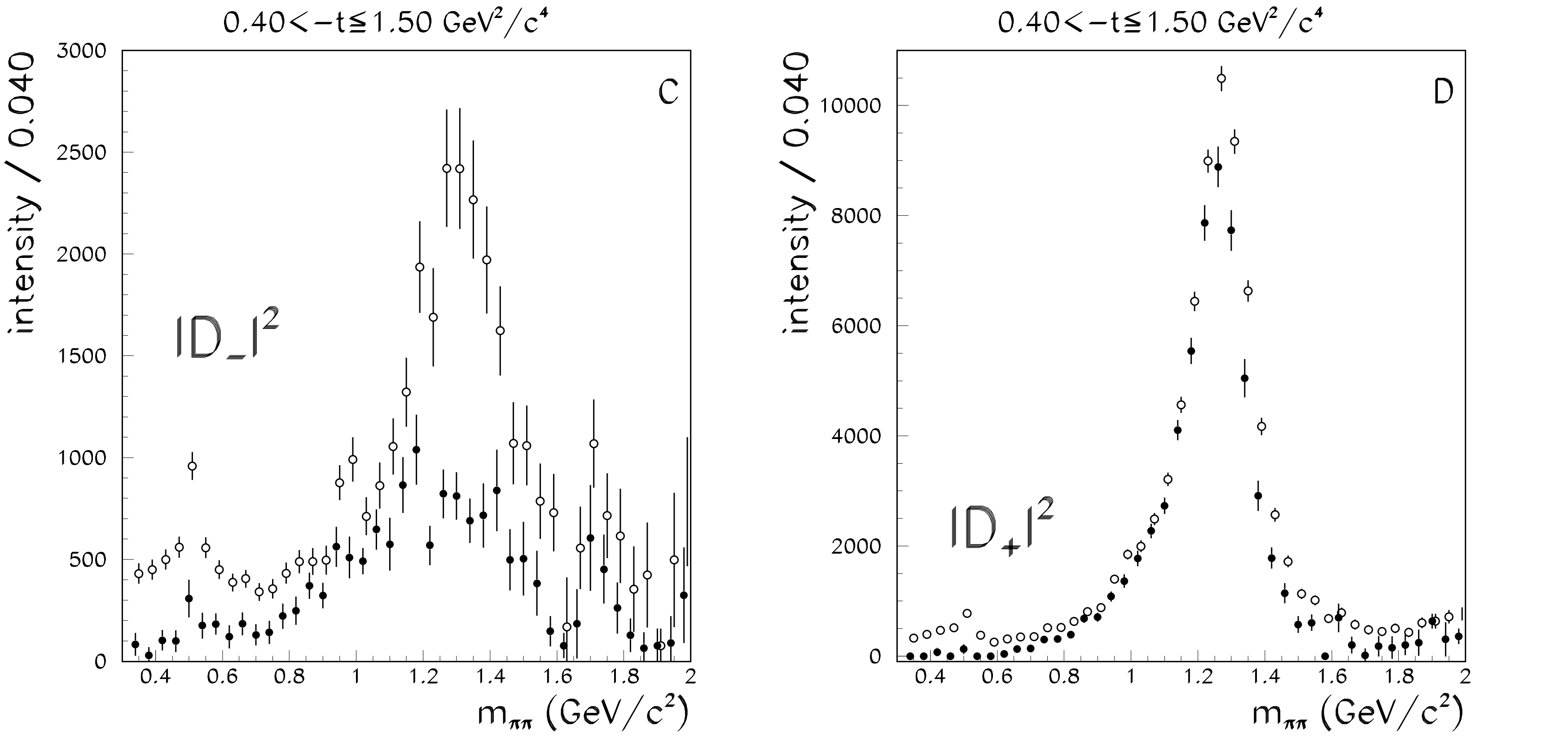
At small t: $|t| < 0.1$:





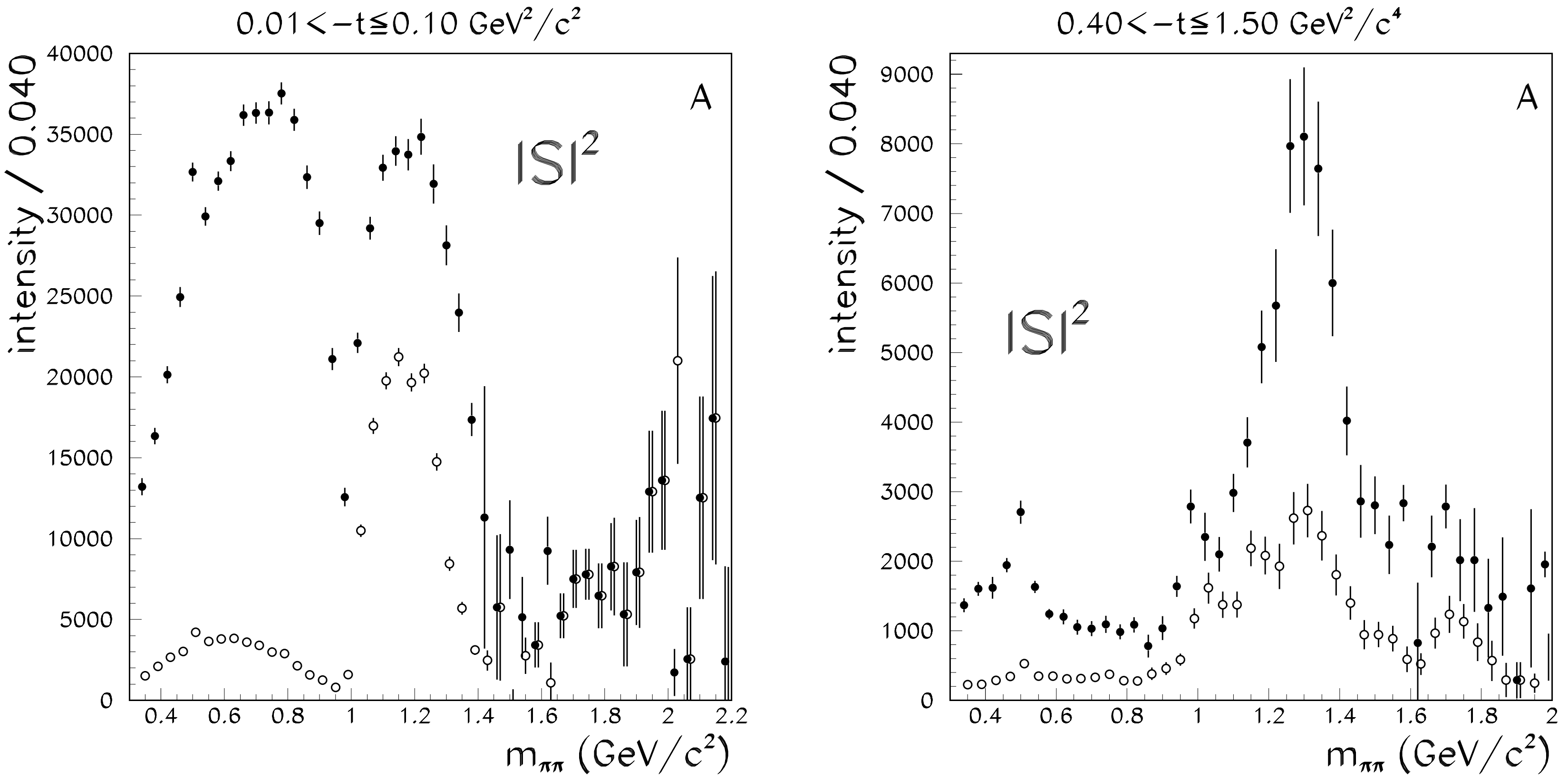
BNL analysis

At large t: $|t| > 0.4$:

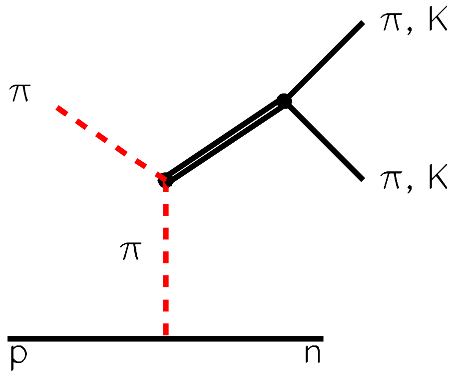


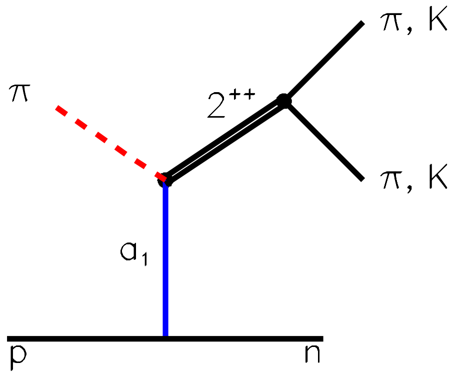
BNL analysis

The S-wave has a very prominent structure at large $|t|$.



Reggezied exchanges:





$$A^{(\mathrm{pion\,trajectories})}_{\pi p\to\pi\pi n} = \sum_{\pi_j} A(\pi\pi_j \to \pi\pi) R_{\pi_j}(s_{\pi N}, q^2)\left(\varphi_n^+(\vec{\sigma}\vec{p}_\perp)\varphi_p\right) g^{(\pi_j)}_{pn}\,.$$

$$A^{(a_1-\mathrm{trajectories})}_{\pi p\to\pi\pi n} = \sum_{a_1^{(j)}} A(\pi a_1^{(j)} \to \pi\pi) R_{a_1^{(j)}}(s_{\pi N}, q^2)\left(\varphi_n^+(\vec{\sigma}\vec{n}_z)\varphi_p\right) g^{(a_{1j})}_{pn}\,.$$

$$R_{\pi_j}(s_{\pi N}, q^2) = \exp\left(-i\frac{\pi}{2}\alpha_\pi^{(j)}(q^2)\right)\frac{\left(s_{\pi N}/s_{\pi N0}\right)^{\alpha_\pi^{(j)}(q^2)}}{\sin\left(\frac{\pi}{2}\alpha_\pi^{(j)}(q^2)\right)\Gamma\left(\frac{1}{2}\alpha_\pi^{(j)}(q^2)+1\right)}$$

$$R_{a_1^{(j)}}(s_{\pi N}, q^2) = i\exp\left(-i\frac{\pi}{2}\alpha_{a_1}^{(j)}(q^2)\right)\frac{\left(s_{\pi N}/s_{\pi N0}\right)^{\alpha_{a_1}^{(j)}(q^2)}}{\cos\left(\frac{\pi}{2}\alpha_{a_1}^{(j)}(q^2)\right)\Gamma\left(\frac{1}{2}\alpha_{a_1}^{(j)}(q^2)+\frac{1}{2}\right)}$$

Features of reggezied a_1 exchange:

$$A(\pi a_1^{(j)} \to \pi\pi) = \sum_J \epsilon_\beta^{(-)} \left[A^{(J+)}_{\pi a_1^{(j)} \to \pi\pi} X^{(J+1)}_{\beta\mu_1\ldots\mu_J} + A^{(J-)}_{\pi a_1^{(j)} \to \pi\pi} Z^{\beta}_{\mu_1\ldots\mu_J} \right] X^{(J)}_{\nu_1\ldots\nu_J} \,,$$

$$A(\pi a_1^{(k)} \to \pi\pi) = \sum_J \alpha_J |\vec{p}|^{J-1} |\vec{k}|^J \left(W_0^{(J)} Y_J^0(\Theta,\varphi) + W_1^{(J)} Re Y_J^1(\Theta,\varphi) \right)$$

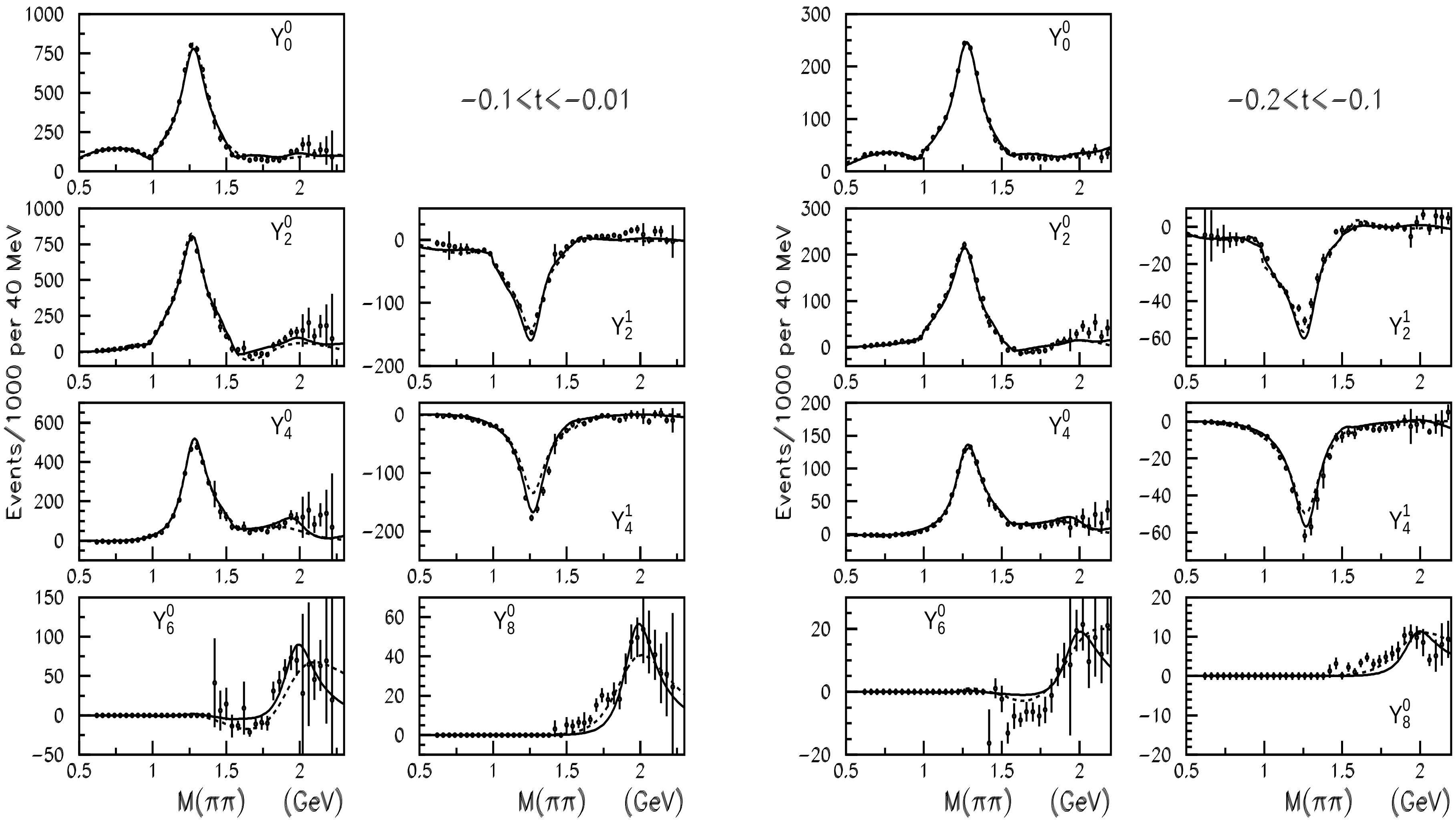
where:

$$\begin{aligned} W_{0k}^{(J)} &= -N_{J0} \left(k_{3z} - \frac{|\vec{p}|}{2} \right) \left(|\vec{p}|^2 A^{(J+)}_{\pi a_1^{(k)} \to \pi\pi} - A^{(J-)}_{\pi a_1^{(k)} \to \pi\pi} \right) \\ W_{1k}^{(J)} &= -\frac{N_{J1}}{J(J+1)} k_{3x} \left(|\vec{p}|^2 J\, A^{(J+)}_{\pi a_1^{(k)} \to \pi\pi} + (J+1) A^{(J-)}_{\pi a_1^{(k)} \to \pi\pi} \right) \end{aligned} \qquad (1)$$

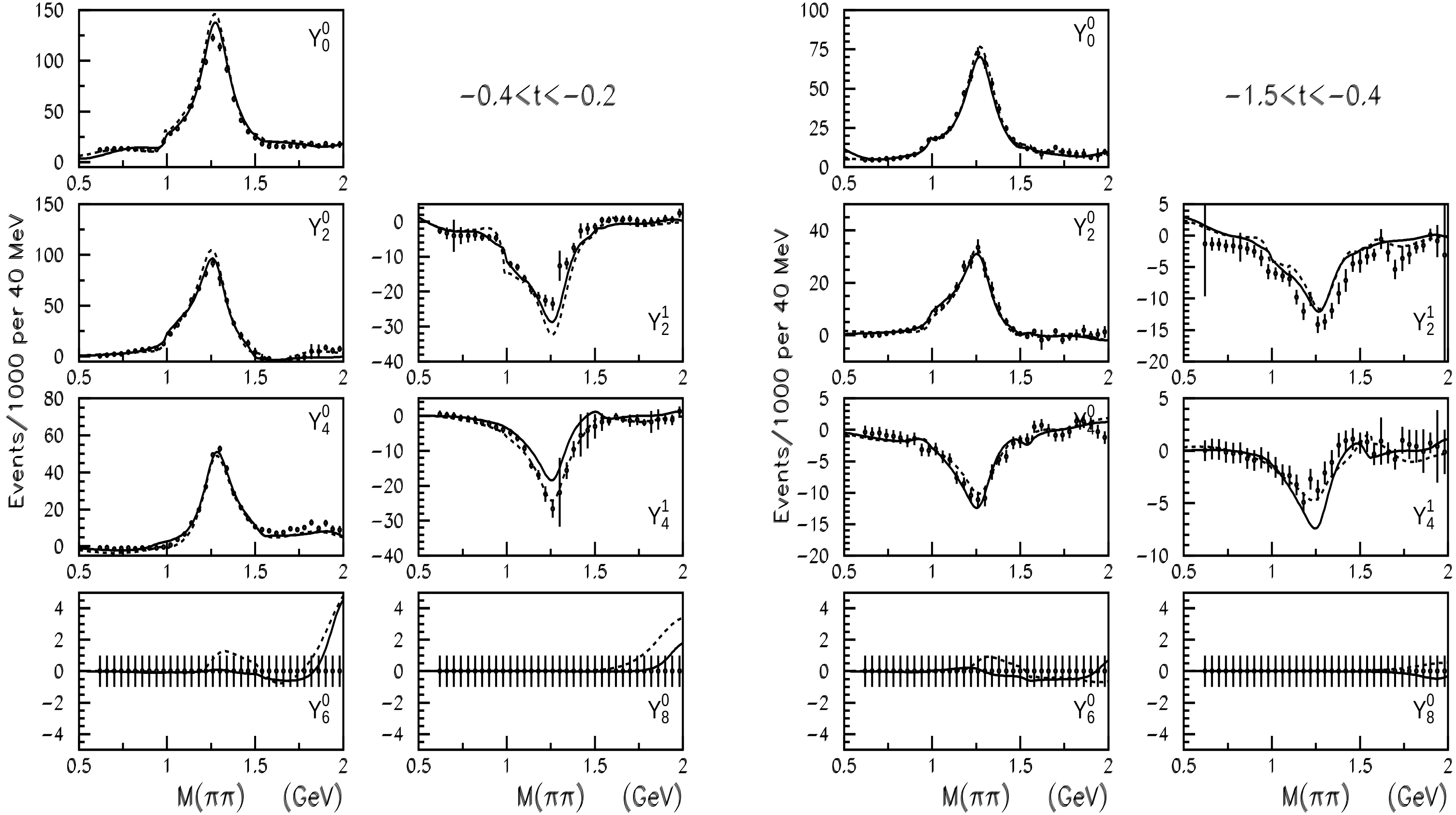
Then $< Y_J^2 >$ moments in the cross section are $(k_{3x}/k_{3z})^2$.

However the contribution to $< Y_J^0 >$ could be rather large already at small t.

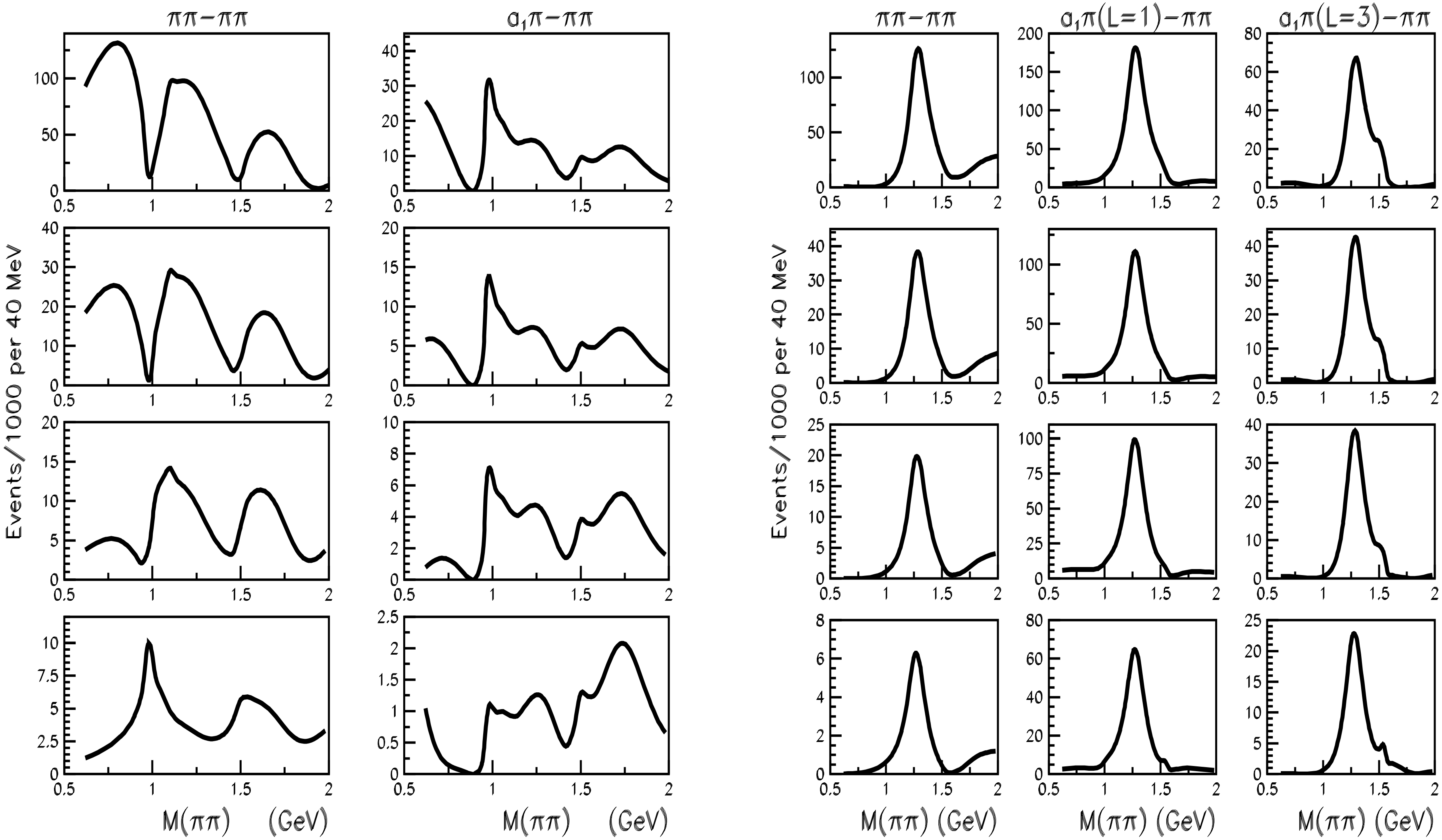
The description of $\pi N \to \pi^0\pi^0 N$ (E852)



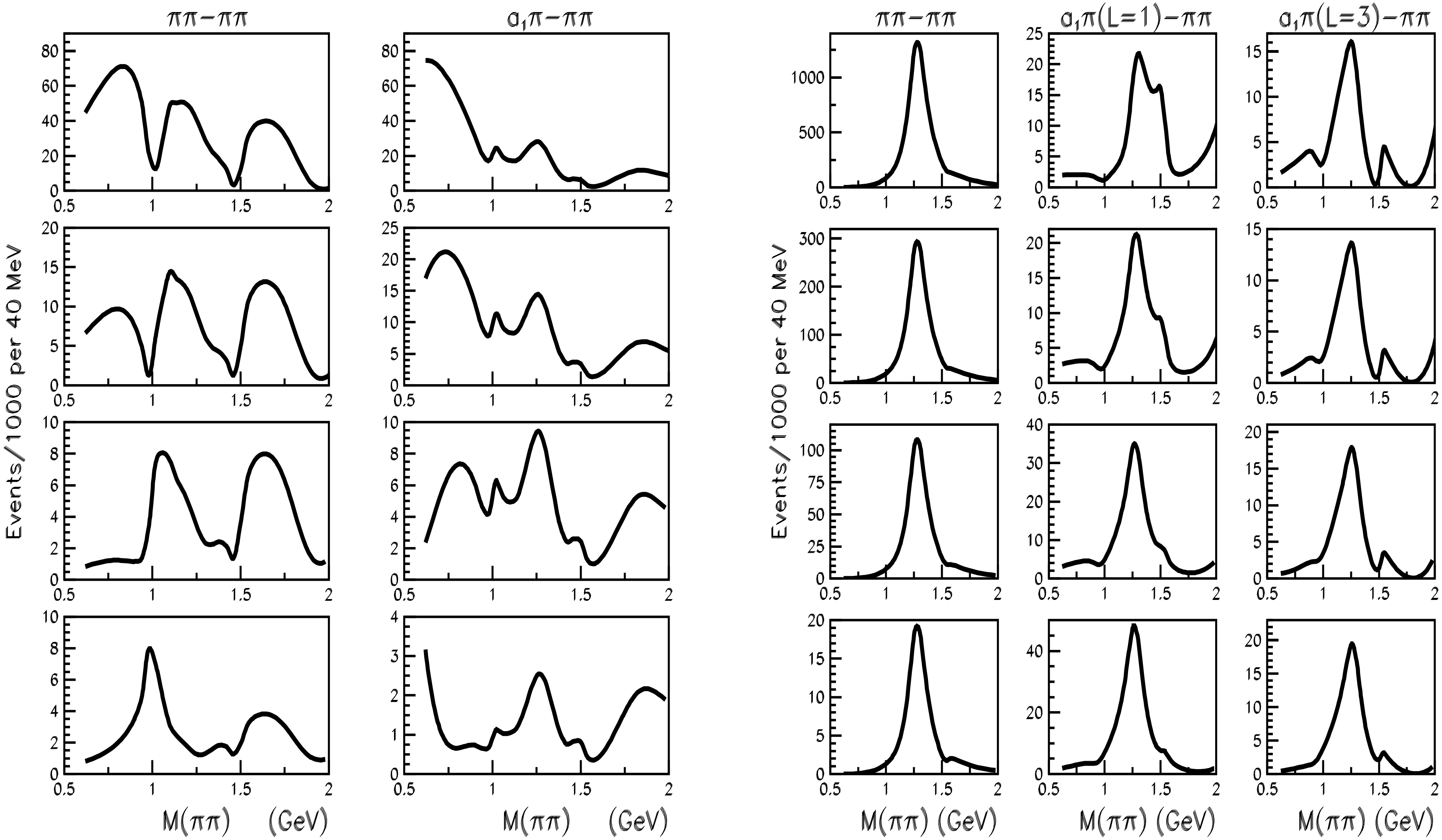
The description of $\pi N \to \pi^0 \pi^0 N$ (E852)



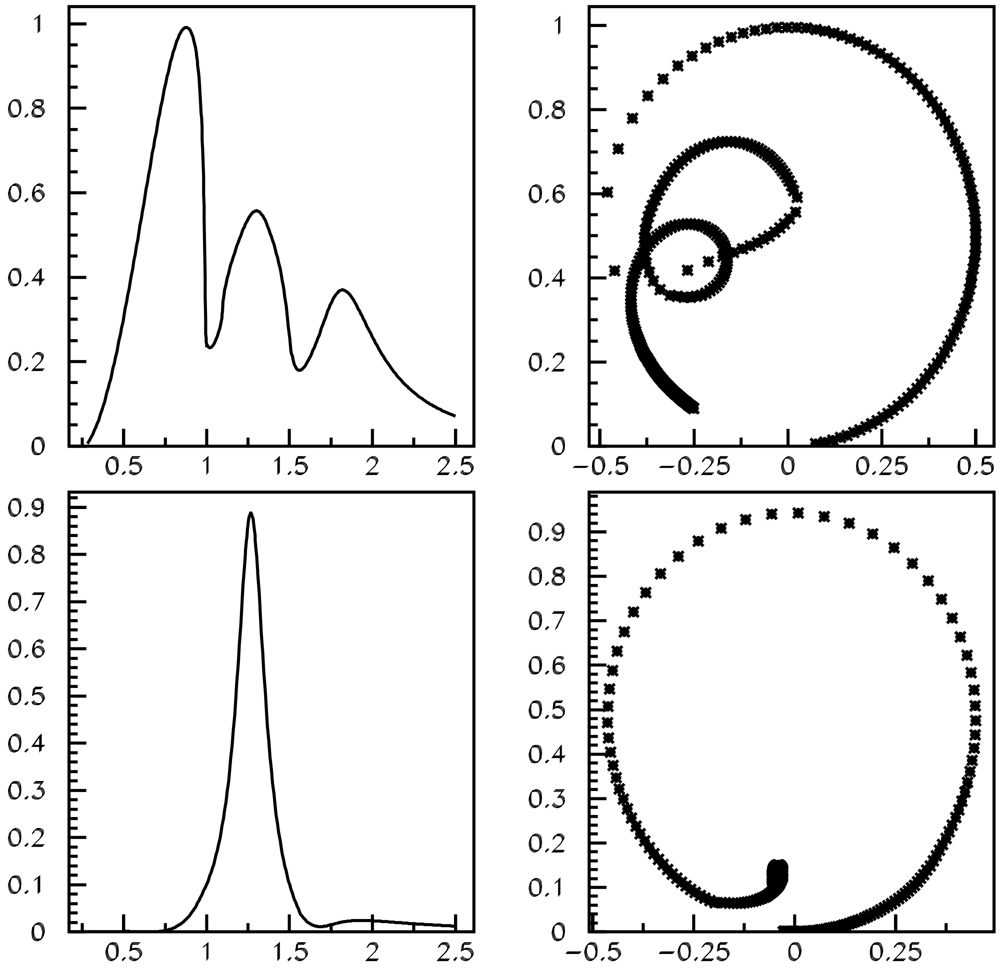
S and D-waves at different t-intervals. Solution 1



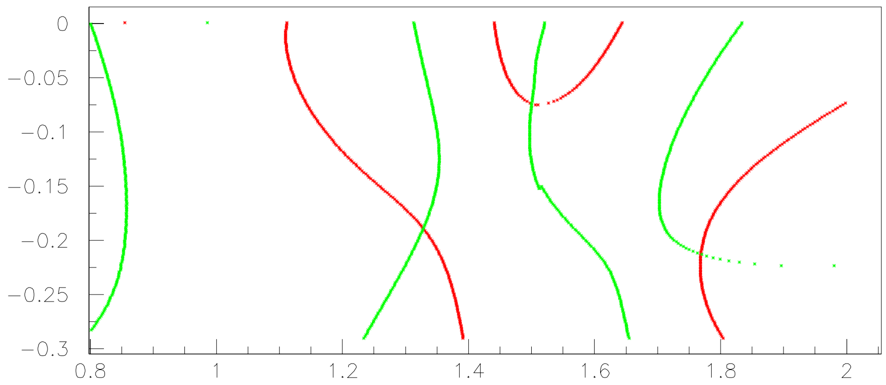
S and D-waves at different t-intervals. Solution 2

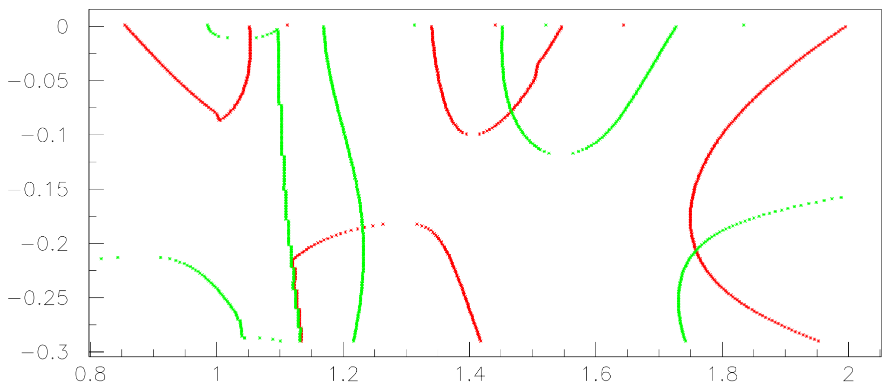


The S and D-wave $\pi\pi \to \pi\pi$ amplitudes



The S-wave pole position in the complex S-plane





		Pole position
$f_0(980)$	**II sheet**	$1030^{+30}_{-10} - i(35^{+10}_{-16})$
$f_0(1370)$	**IV sheet**	$1290 \pm 50 - i(170^{+20}_{-40})$
$f_0(1500)$	**IV sheet**	$1486 \pm 10 - i(57 \pm 5)$
$f_0(1530)$	**IV sheet**	$1510 \pm 130 - i(800^{+100}_{-150})$
$f_0(1750)$	**V sheet**	$1800 \pm 60 - i(200 \pm 40)$

Isoscalar $J^{PC} = 2^{++}$ sector

	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$
M	1.286 ± 0.025	1.540 ± 0.015	1.560 ± 0.020	$2.200^{+0.300}_{-0.200}$
$g^{(\alpha)}_{\pi\pi}$	0.920 ± 0.020	-0.05 ± 0.080	0.280 ± 0.100	-0.30 ± 0.15
$g^{(\alpha)}_{\eta\eta}$	0.420 ± 0.060	0.27 ± 0.15	0.400 ± 0.200	$1.2 \pm 0.6^*$
$g^{(\alpha)}_{4\pi}$	-0.150 ± 0.200	0.370 ± 0.150	1.170 ± 0.450	1.0 ± 0.4
$g^{(\alpha)}_{\omega\omega}$	0^*	0^*	0.540 ± 0.150	-0.05 ± 0.2
	Pole position			
III sheet	1.270 ± 0.008	1.530 ± 0.012		
	$-i\,0.097 \pm 0.008$	**-i** 0.064 ± 0.010		
III sheet			1.690 ± 0.015	
			$-i\,0.290 \pm 0.020$	
IV sheet			1.560 ± 0.015	
			$-i\,0.140 \pm 0.020$	

Isoscalar $J^{PC} = 4^{++}$ state

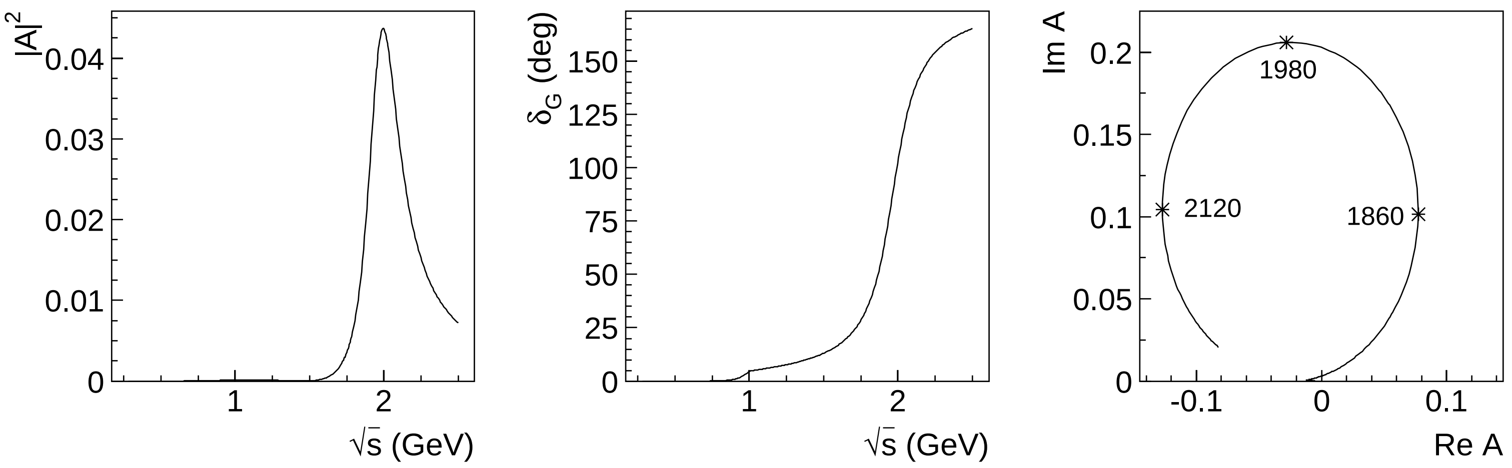
Fitted as two channel ($\pi\pi$ and 4π) one pole K-matrix.

M (GeV)	$g_{\pi\pi}$	$g_{4\pi}$	$f_{\pi\pi\to\pi\pi}$
1.970 ± 30	0.550 ± 0.050	0.490 ± 0.080	-0.025 ± 0.050

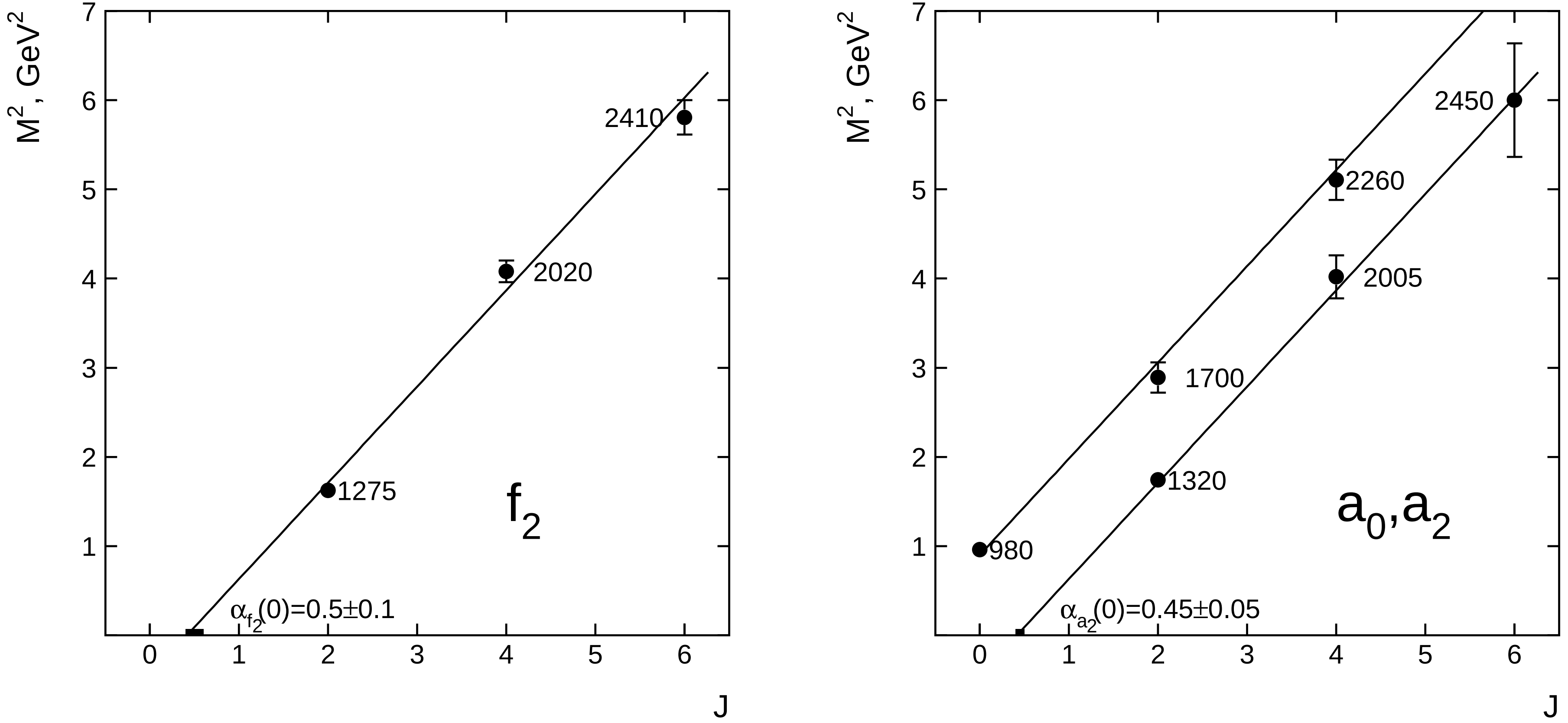
Pole position: $(1966 \pm 25) - i\,(130 \pm 20)$

The $\pi\pi$ amplitude has a peak at 1995 MeV. $Br(\pi\pi) = 20 \pm 3$**%**

PDG: $M = 2020 \pm 10$ **MeV** $Br(\pi\pi) = 17 \pm 1.5$



The f_2 and a_2 Regge trajectories



Fit without $f_0(1370)$

Fit of the BNL data deteriorated everywhere. Largest effect at:

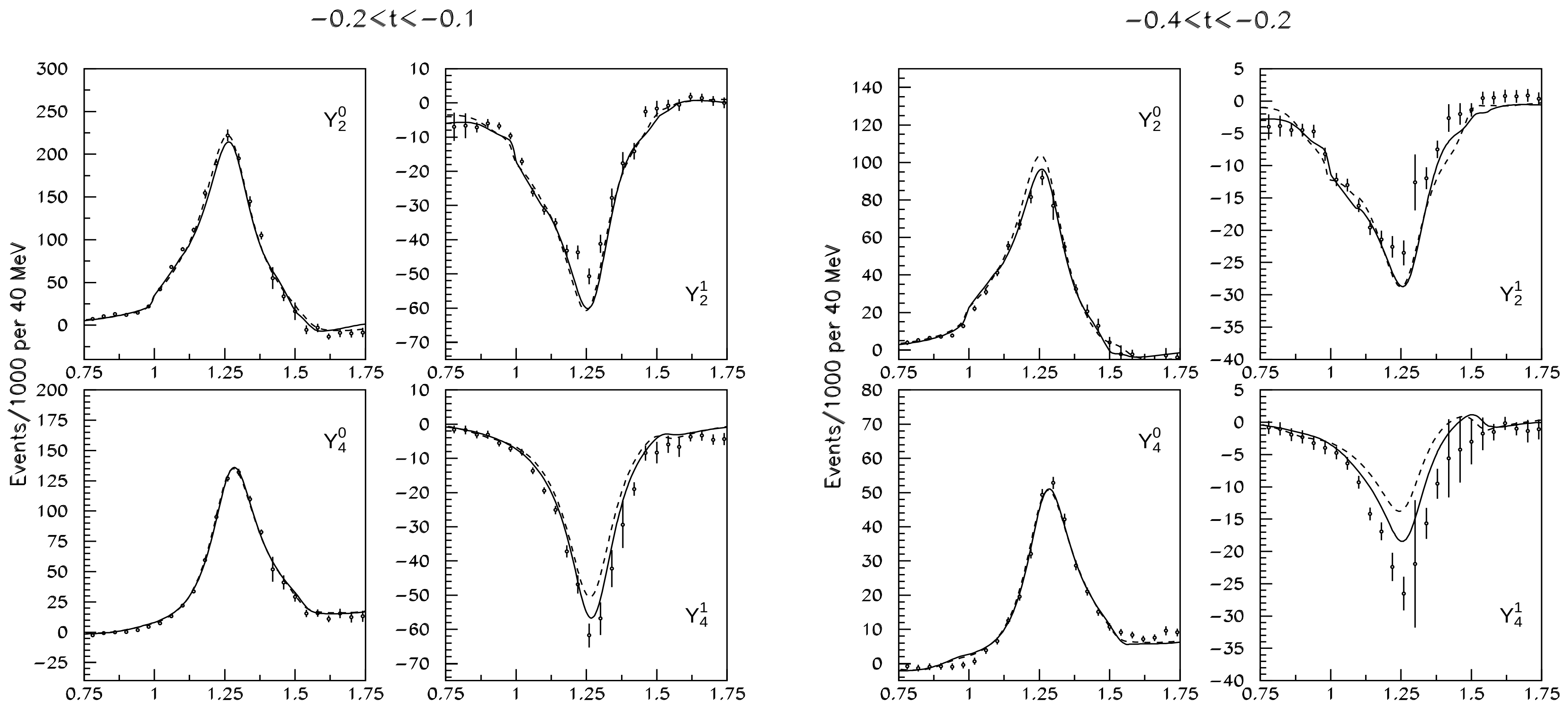
$-0.2 < t < -0.1 \quad 1.84 \to 3.63$

$-0.4 < t < -0.2 \quad 2.07 \to 4.90$

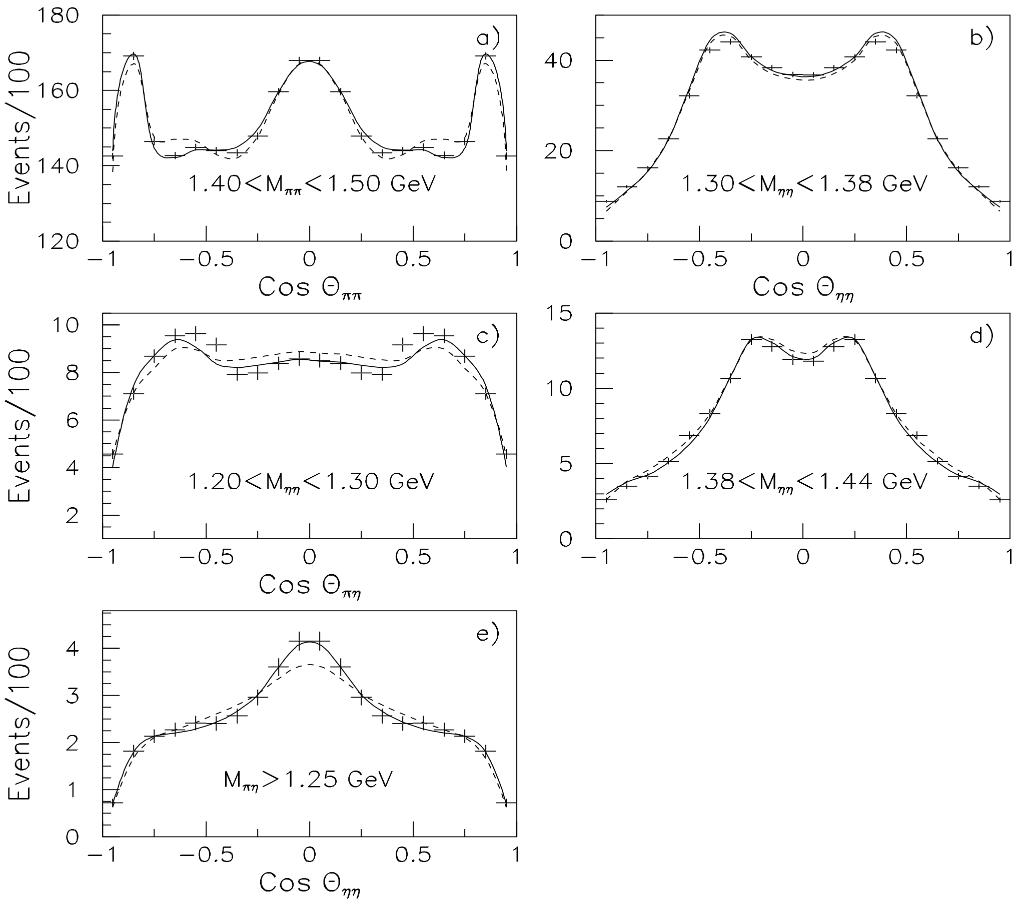
Fit of other data sets:

Data	Solution 1	Solution 2	Solution 2(-) (no $f_0(1370)$)
$\bar{p}p \to \pi^0\pi^0\pi^0$ (Liq)	1.360	1.356	1.443
$\bar{p}p \to \pi^0\pi^0\pi^0$ (Gas)	1.238	1.242	1.496
$\bar{p}p \to \eta\pi^0\eta$ (Liq)	1.350	1.442	1.446
$\bar{p}p \to \eta\pi^0\eta$ (Gas)	1.503	1.371	1.315
$\bar{p}p \to \pi^0\eta\pi^0$ (Liq)	1.210	1.236	1.412
$\bar{p}p \to \pi^0\eta\pi^0$(Gas)	1.099	1.119	1.227
$\pi\pi \to \eta\eta$ (S-wave)	1.08	1.19	1.38
$\pi\pi \to \eta\eta'$ (S-wave)	0.26	0.41	0.45

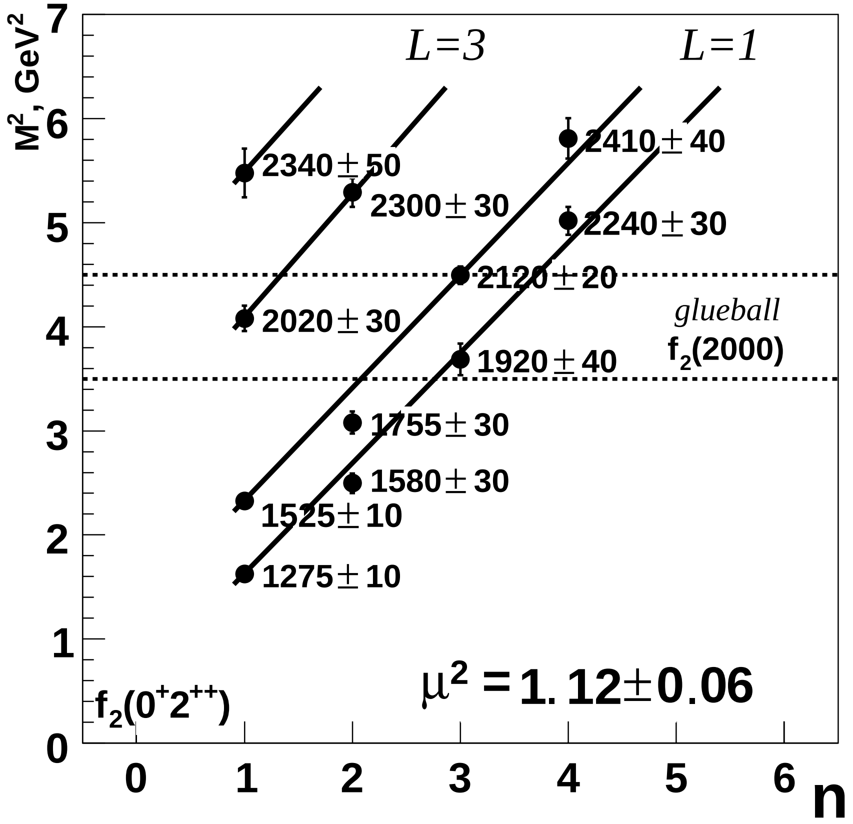
The description of BNL data from Solution 2 (full curves) and from the fit without $f_0(1370)$ (dashed curves)

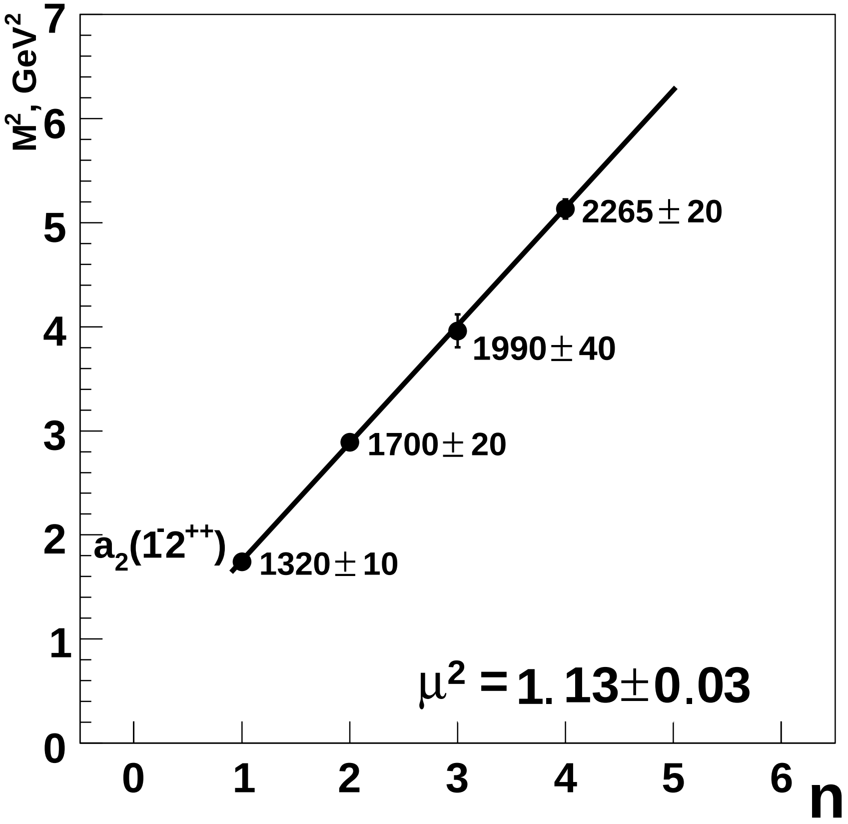


The description of Crystal Barrel data from Solution 2 (full curves) and from the fit without $f_0(1370)$ (dashed curves)



The trajectories of the f_2 and a_2 mesons in (J, M^2) plane





Conclusion

1. **The piN interaction at large pion energies is one of the best source to study resonances in the region 1.7-2.2 GeV and can supply a vital information for the higher energy region.**

2. **The study of t-dependence can provide a new information about resonance properties. However a simplified approach to the data analysis can lead to a misidentification of quantum numbers.**

3. **The reggeon exchange approach is a most suitable tool for analysis of the $\pi N \to mesons N$ data, providing a natural connection of the regions of small and large t.**