Pion FF in QCD SR approach with NonLocal Condensates

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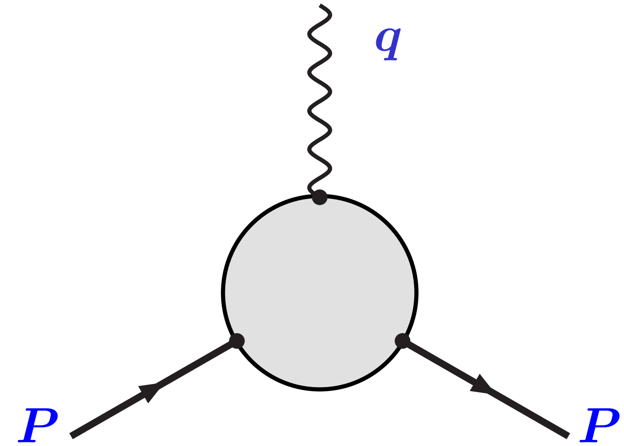
- Definition of pion FF
- AAV correlator and its diagrams
- Introducing NLC in QCD calculations
- NonLocal Condensates QCD SR
- QCD SR vs Experiment
- Conclusion

Definition of pion Form Factor

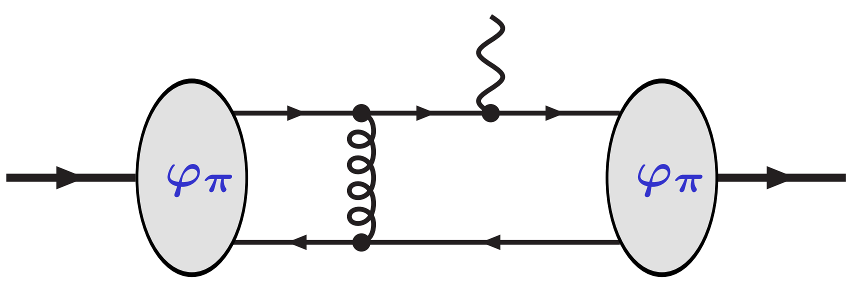
Pion FF F_π is defined by the matrix element

$$\langle \pi^+(P') | J_\mu(0) | \pi^+(P) \rangle = (P + P')_\mu F_\pi(Q^2) ,$$

where J_μ is the electromagnetic current, $(P' - P)^2 = q^2 \equiv -Q^2$ is the photon virtuality, and pion FF is normalized to $F_\pi(0) = 1$.



At asymptotically large Q^2, the pQCD factorization gives the pion FF



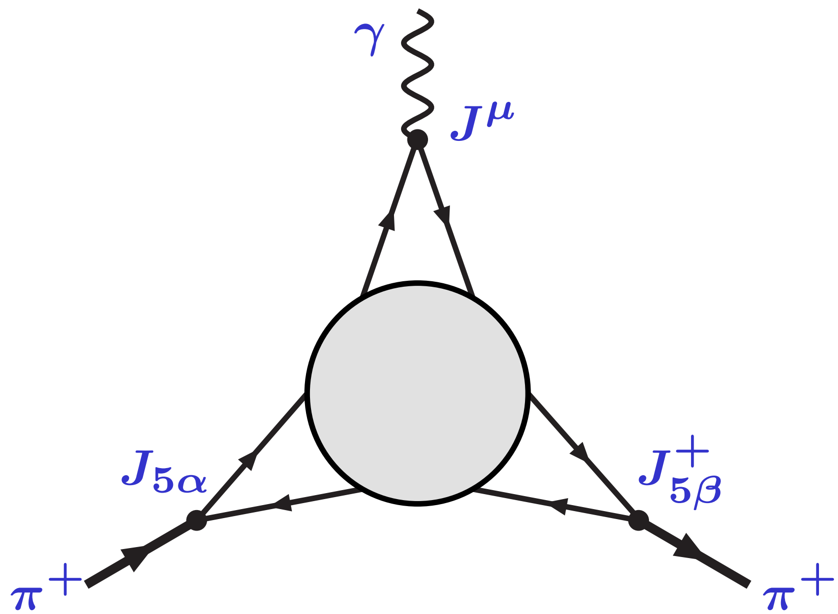
$$F_\pi(Q^2) = \frac{8\pi\alpha_s(Q^2) f_\pi^2}{9Q^2} \left| \int_0^1 \frac{\varphi_\pi(x, Q^2)}{x} dx \right|^2$$

in terms of the pion DA $\varphi_\pi(x, Q^2)$ of the leading twist.

AAV correlator

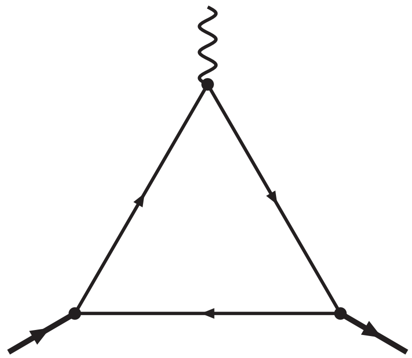
Axial-Axial-Vector correlator can be used for studying pion FF by QCD SR technique:

$$\int\!\!\int d^4x\, d^4y\, e^{i(qx-p_2y)} \langle 0|T\left[J^{+}_{5\beta}(y) J^{\mu}(x) J_{5\alpha}(0)\right]|0\rangle$$



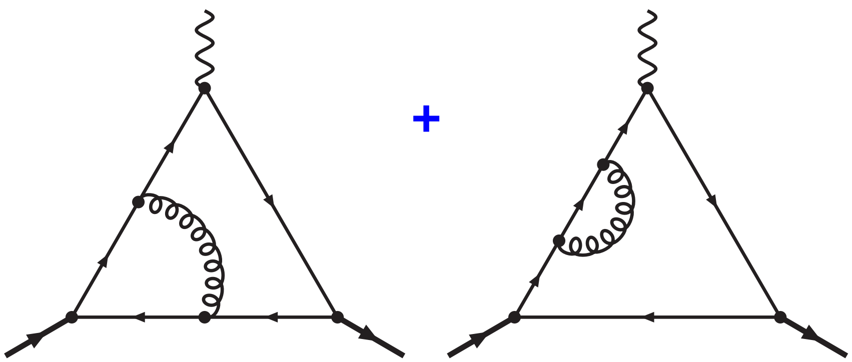
where EM current $J^{\mu}(x) = e_u\, \overline{u}(x)\gamma^{\mu}u(x) + e_d\, \overline{d}(x)\gamma^{\mu}d(x)$ and axial-vector current: $J_{5\alpha}(x) = \overline{d}(x)\gamma_5\gamma_{\alpha}u(x)$.

Diagramms for AAV-correlator



1 Perturbative LO term

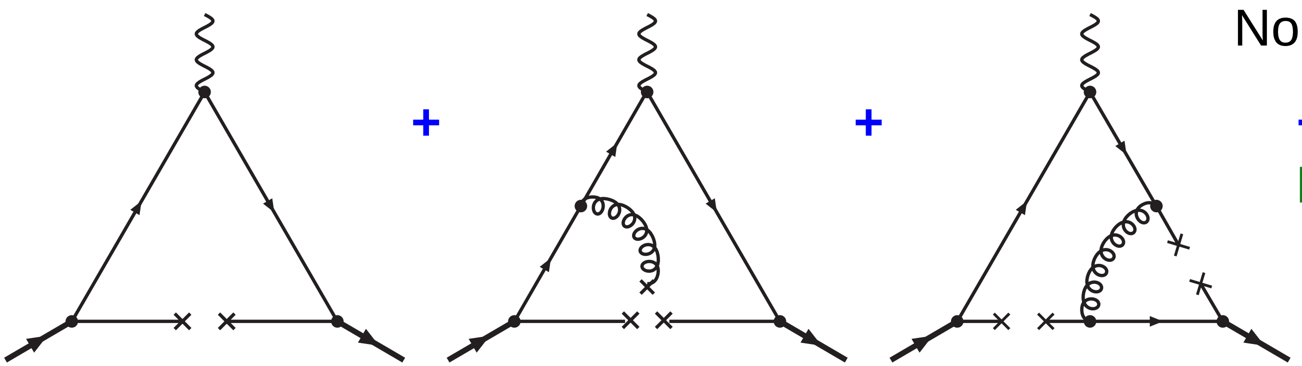
Ioffe&Smilga 1982



Perturbative NLO terms

+ . . .

Braguta&Onishchenko 2004



Nonperturbative terms

+ . . .

Nesterenko&Radyushkin

⊕**Ioffe&Smilga 1982**

local condensates

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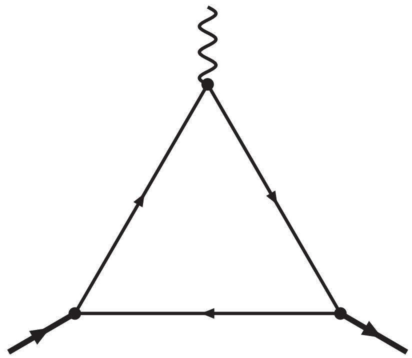

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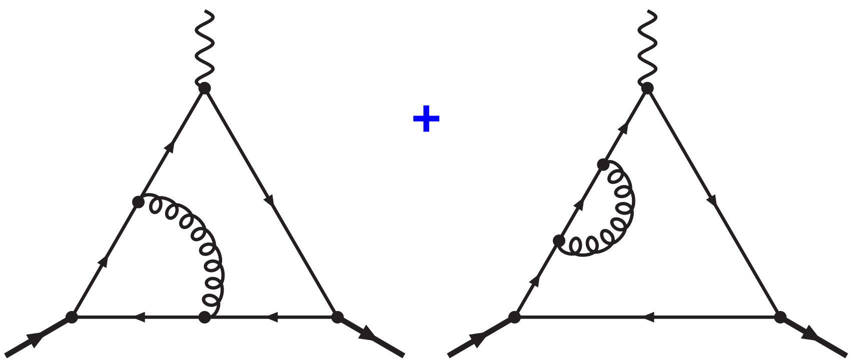

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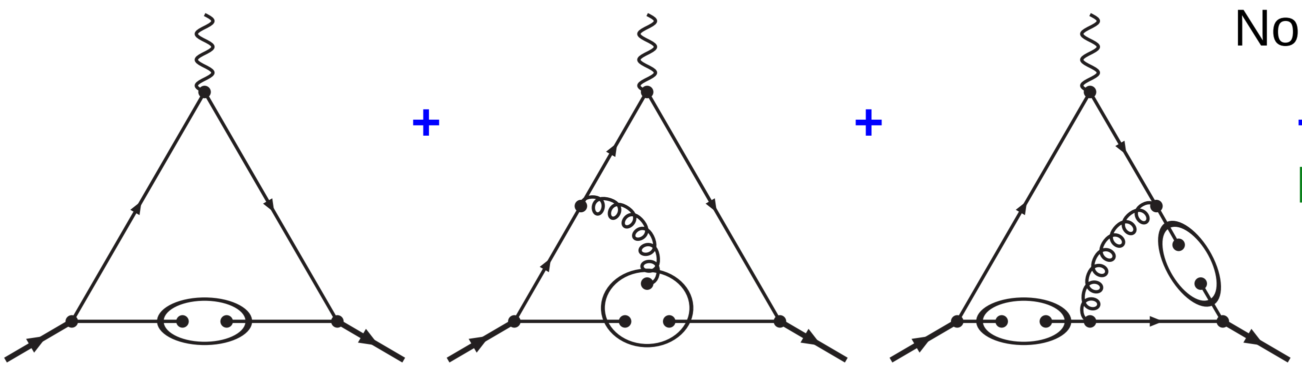


Nonperturbative terms

+ . . .

Bakulev&Radyushkin 1991

nonlocal condensates

Introducing NLC in QCD calculations

$$T\left(\overline{\psi}\psi\right) = \overline{\psi}\psi \quad + \quad :\overline{\psi}\psi: \text{ (Wick theorem)}$$

$$\langle 0|T\left(\overline{\psi}\psi\right)|0\rangle = i^{-1}\hat{S}_0(x) \quad + \quad ?$$

QCD PT

$$\langle :\overline{\psi}\psi:\rangle \stackrel{\text{def}}{=} 0$$

QCD SR

$$\langle :\overline{\psi}(0)\psi(0):\rangle = \langle \overline{q}q\rangle$$

CONST $\neq 0$

[SVZ'79]

Condensate

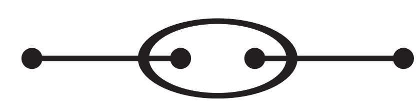
Decay constants,

masses of hadrons

NLC QCD SR

$$\langle :\overline{\psi}(0)\psi(z):\rangle$$

$$F_S(z^2) + \hat{z}F_V(z^2)$$



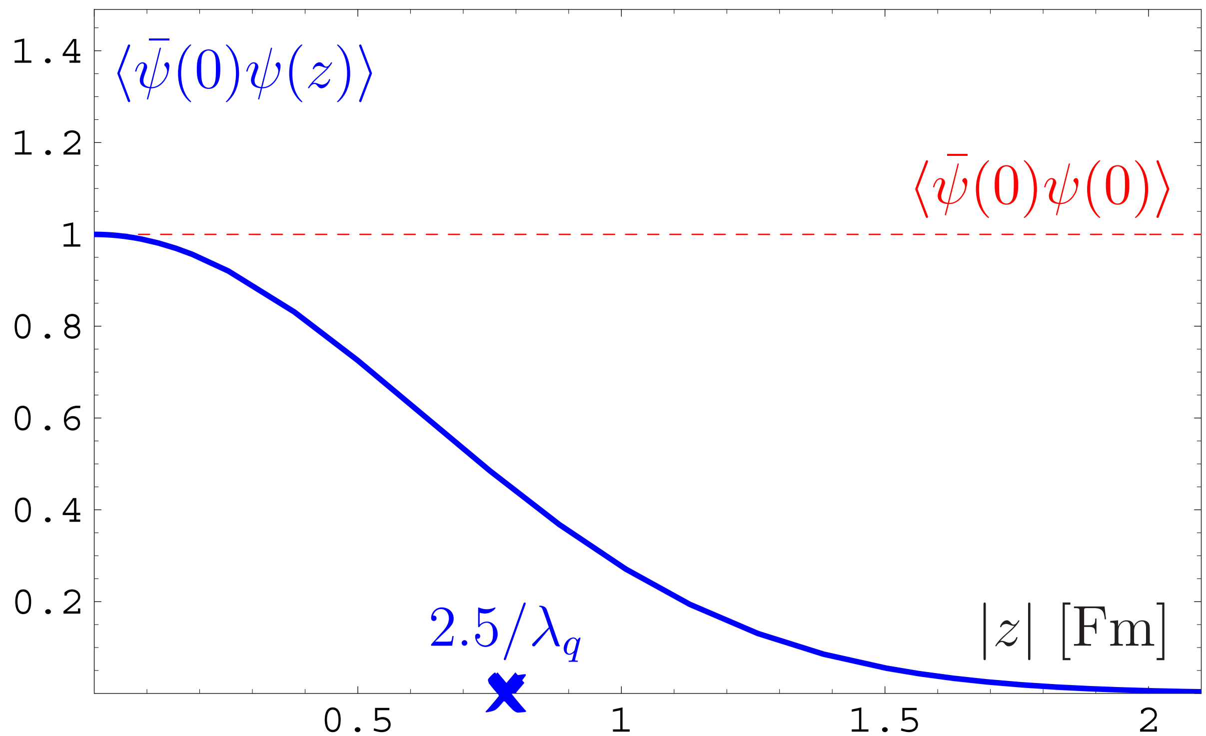
M&R '86

Nonlocal condensate

Distribution Amplitudes,

Form Factors

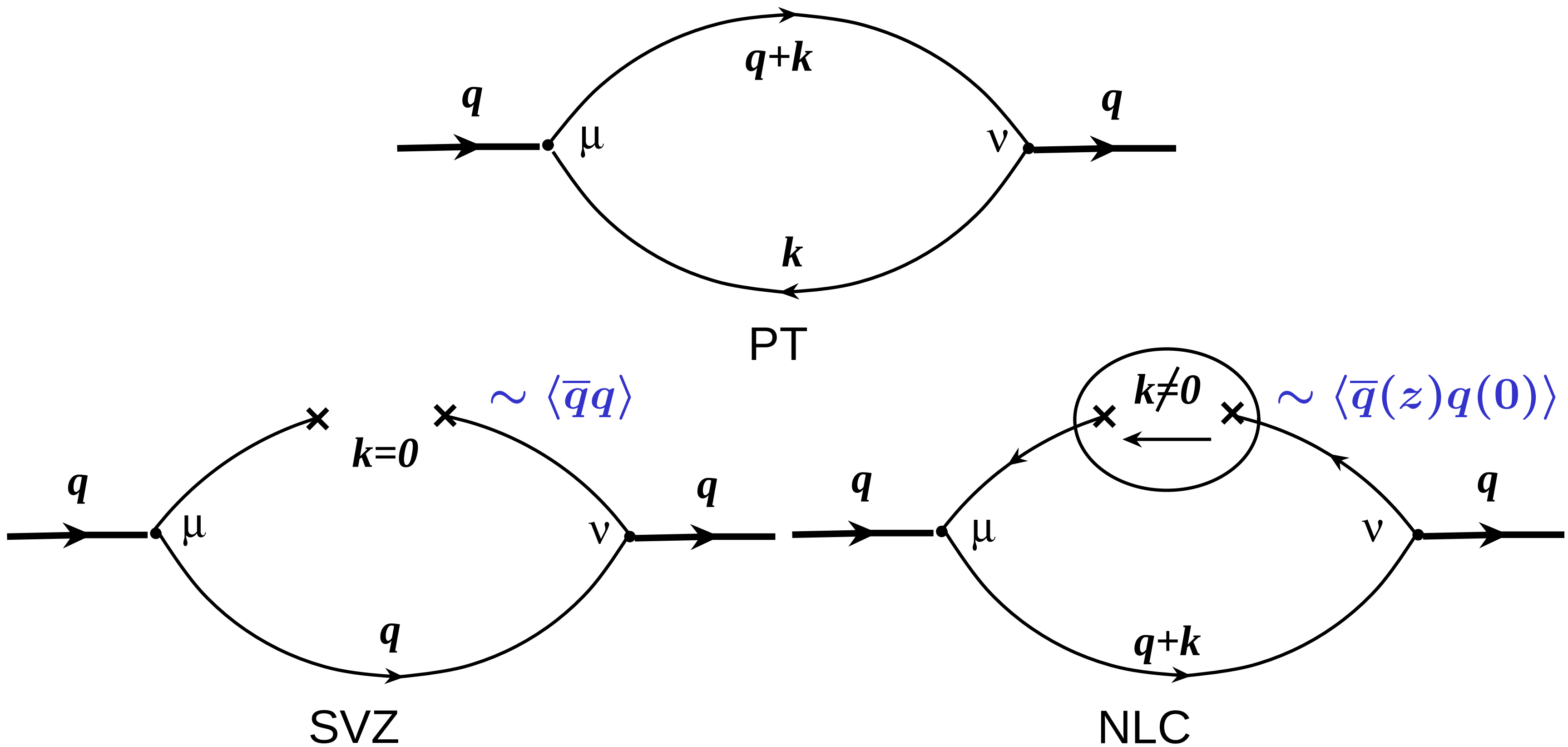
Lattice data of Pisa group



Nonlocality of quark condensates from lattice data of Pisa group in comparison with **local limit**.

Even at $|z| \simeq 0.5$ Fm nonlocality is quite important!

Diagrams for $\langle T\left(J_1(z)J_2(0)\right)\rangle$



Quarks run through vacuum with nonzero momentum $k \neq 0$:

$$\langle k^2\rangle = \frac{\langle\overline{\psi}D^2\psi\rangle}{\langle\overline{\psi}\psi\rangle} = \lambda_q^2 = 0.4 - 0.5\,\text{GeV}^2$$

NLC parameterization

Parameterization for scalar condensate was suggested in works of Bakulev, Mikhailov and Radyushkin:

$$\langle : \overline{\psi}(0)\psi(x) : \rangle \;=\; \langle \overline{\psi}\psi \rangle \int_0^\infty \boxed{f_S(\alpha)}\, e^{\alpha x^2/4}\, d\alpha\,,$$ where $x^2 < 0$.

First approximation which takes into account finite width of quark

distribution in vacuum: $$f_S(\alpha) = \delta\left(\alpha - \frac{\lambda_q^2}{2}\right)\,, \quad \lambda_q^2 = \frac{\langle \overline{\psi} D^2 \psi \rangle}{\langle \overline{\psi}\psi \rangle}\,.$$

Such representation corresponds to **Gaussian** form $\sim \mathbf{exp}\left(\lambda_q^2 x^2/8\right)$ of NLC in coordinate representation.

The **smooth model** $f_S(\alpha) \sim \alpha^{n-1}\mathbf{exp}\left(-\Lambda^2/\alpha - \sigma^2\alpha\right)$ has a sensible asymptotic form $\langle \overline{\psi}(0)\psi(x) \rangle \Big|_{x^2 \to \infty} \sim \mathbf{exp}\left(-\Lambda x\right)$ in x-representation.

NLC QCD SR

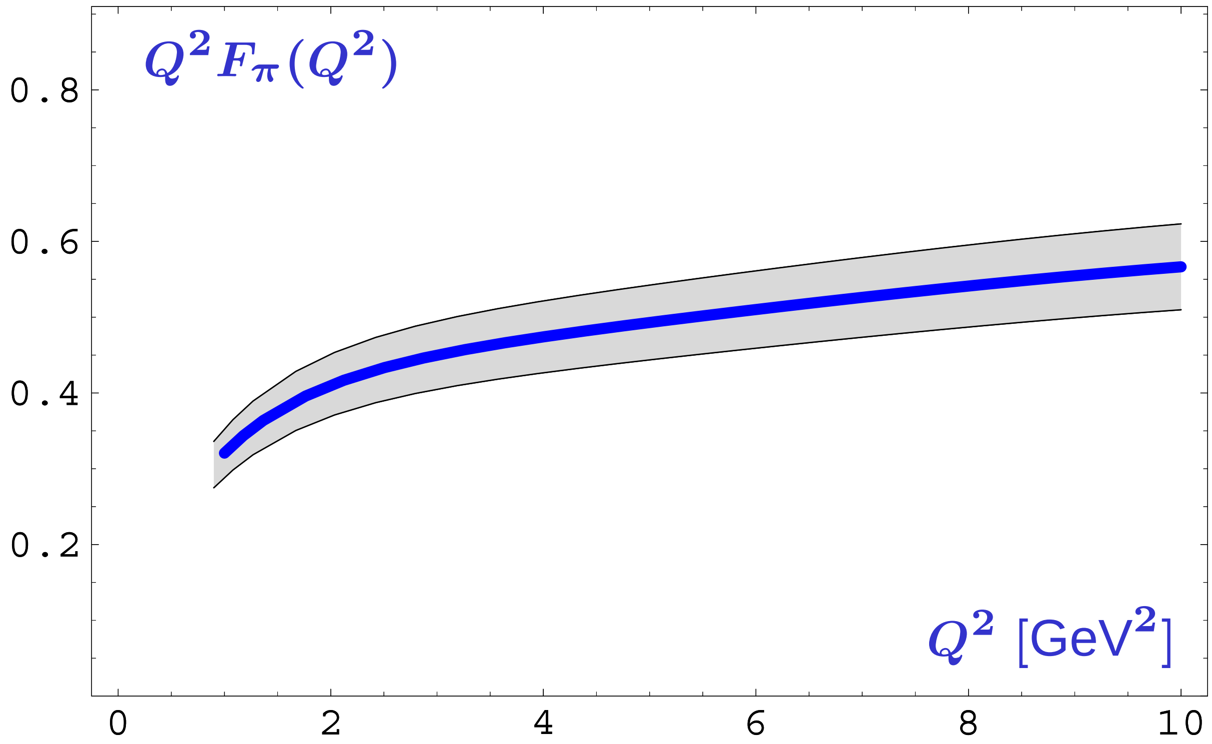
The Borel SR for the pion FF based on three-point AAV correlator:

$$f_\pi^2 F_\pi(Q^2) = \iint\limits_0^{s_0} ds_1\, ds_2\, \rho_3(s_1, s_2, Q^2)\, e^{-(s_1+s_2)/M^2} + \Phi_{\text{OPE}}(Q^2, M^2)\,.$$

Approach	Acc	Condensates	Q^2-behavior of Φ_{OPE}
N&R, I&S 82	LO	local	const $+ Q^2$
B&R 91	LO	local $+$ nonlocal	(const $+ Q^2)(e^{-Q^2\lambda_q^2} +$ const)
B&O 04 - LD	NLO	NO $M^2 \to 0$	$\Phi_{\text{OPE}} \to 0$, $s_0 =$? (f_π LD SR)
Here	NLO	nonlocal	(const $+ Q^2)\, e^{-Q^2\lambda_q^2}$

- Nonlocality improves Q^2 behavior of OPE and as a result widens region of applicability up to $Q^2 \simeq 10$ **GeV**^{2}.
- We use model-independent expression for Φ_{OPE}-term obtained by **Bakulev&Radyushkin**, but significantly different model of condensate's nonlocality.

NLC QCD SR Result



curve	approach
	QCD SR
	LD SR
	ADS/QCD
	Here
▲	**[Cornell 78]**
♦	**[JLab 08]**

Pion FF in standard QCD SR **(red dashed line)** **[N&R⊕I&S 82]**, in Local Duality QCD SR **[B&O 04]** **(dark dashed line)**, ADS/QCD **[Brodsky 07]** **(green dashed line)**, and SR with nonlocal condensates NLC **(blue solid line)** in comparison with recent **[JLab 08]** (♦) and **[Cornell 78]** (▲

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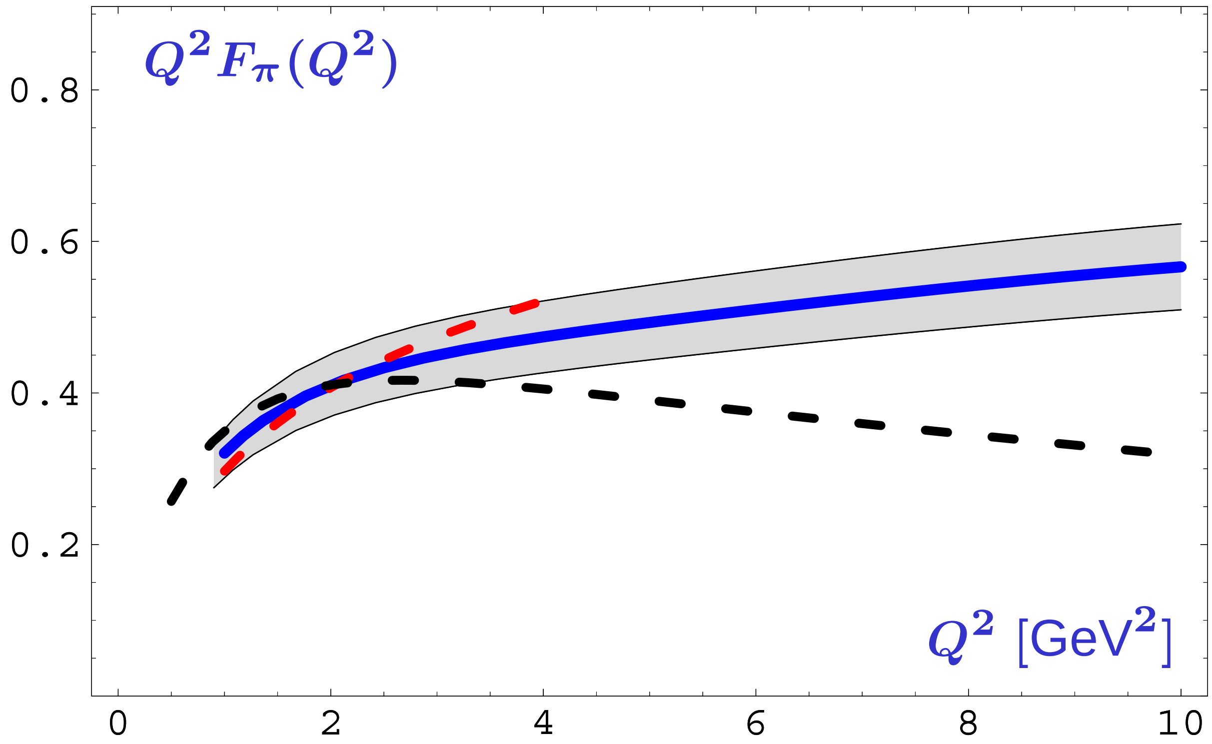

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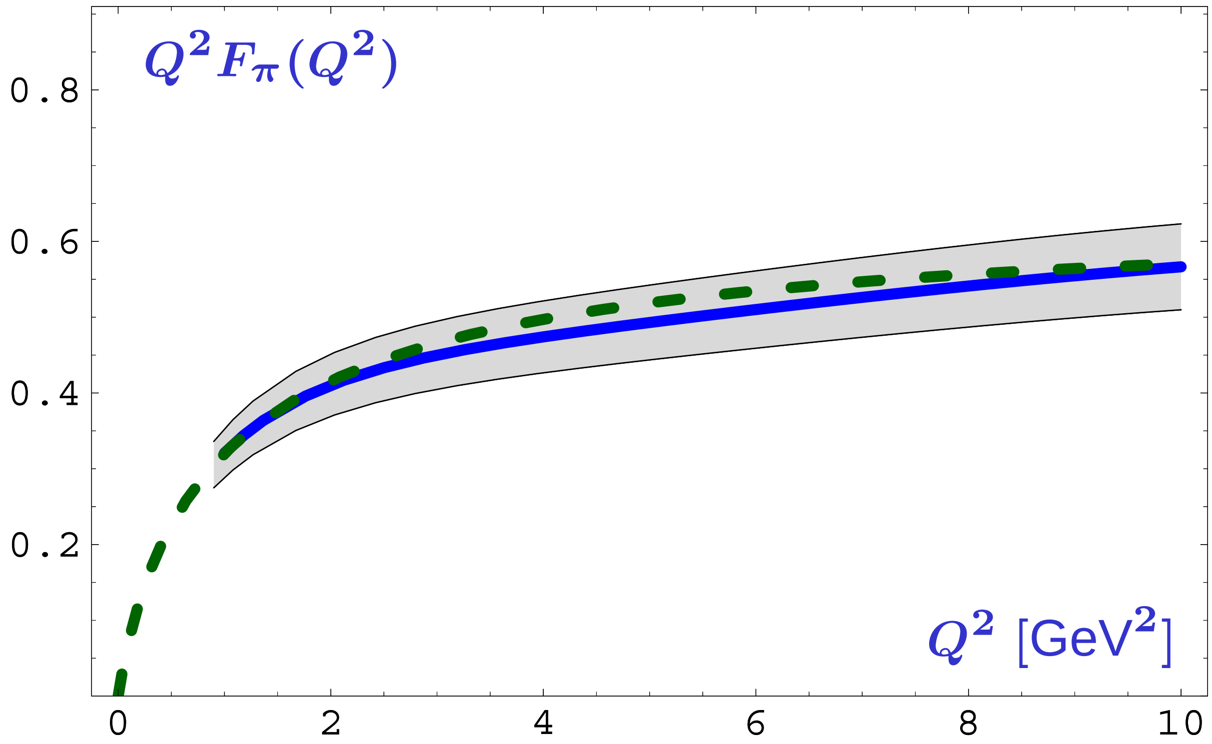

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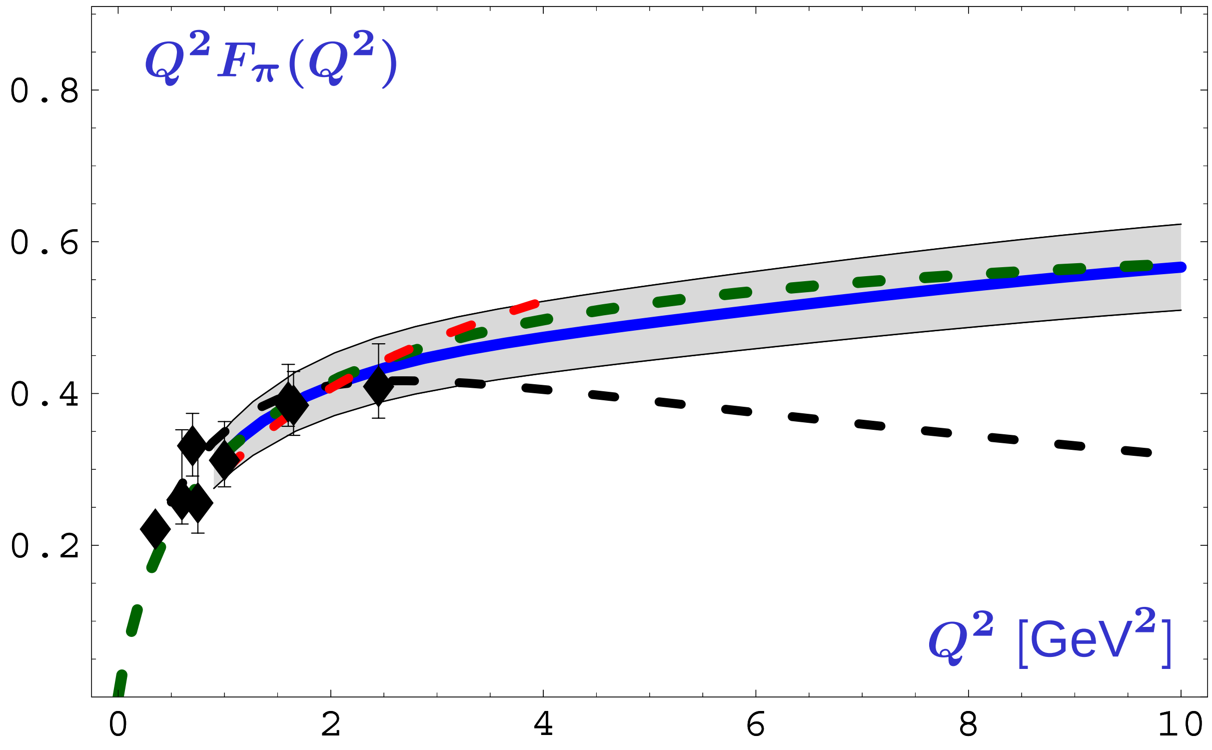

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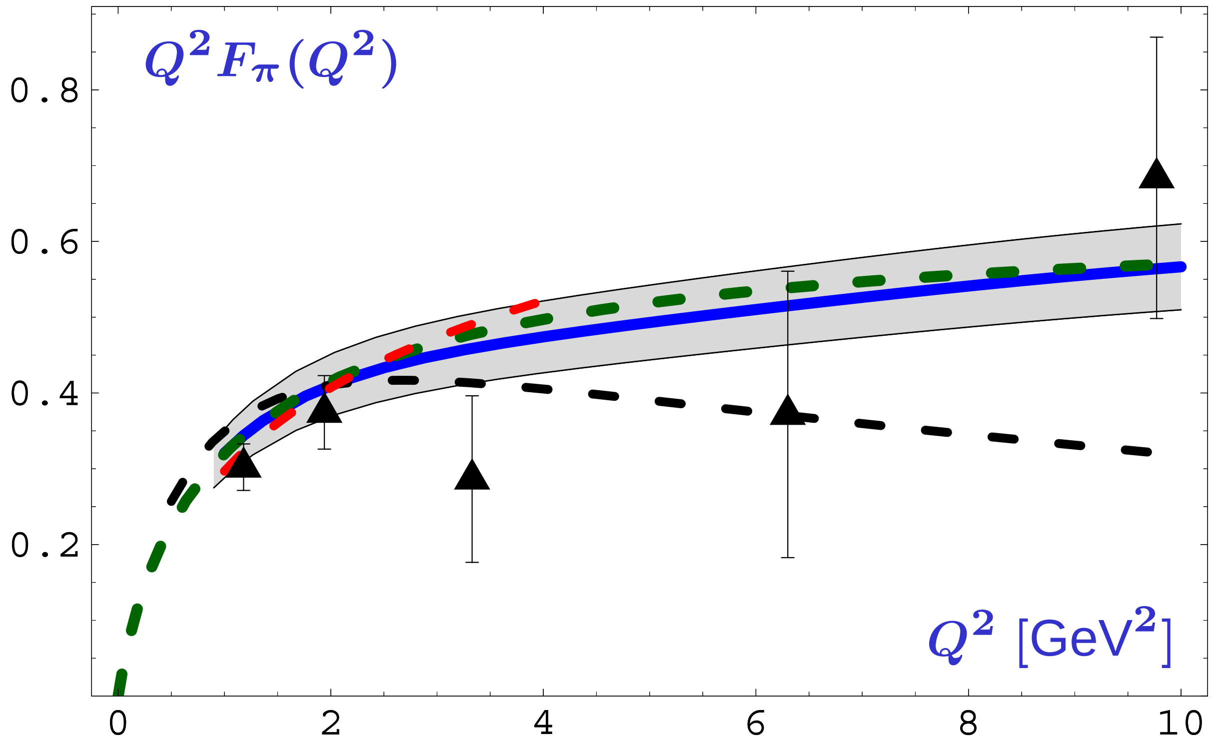

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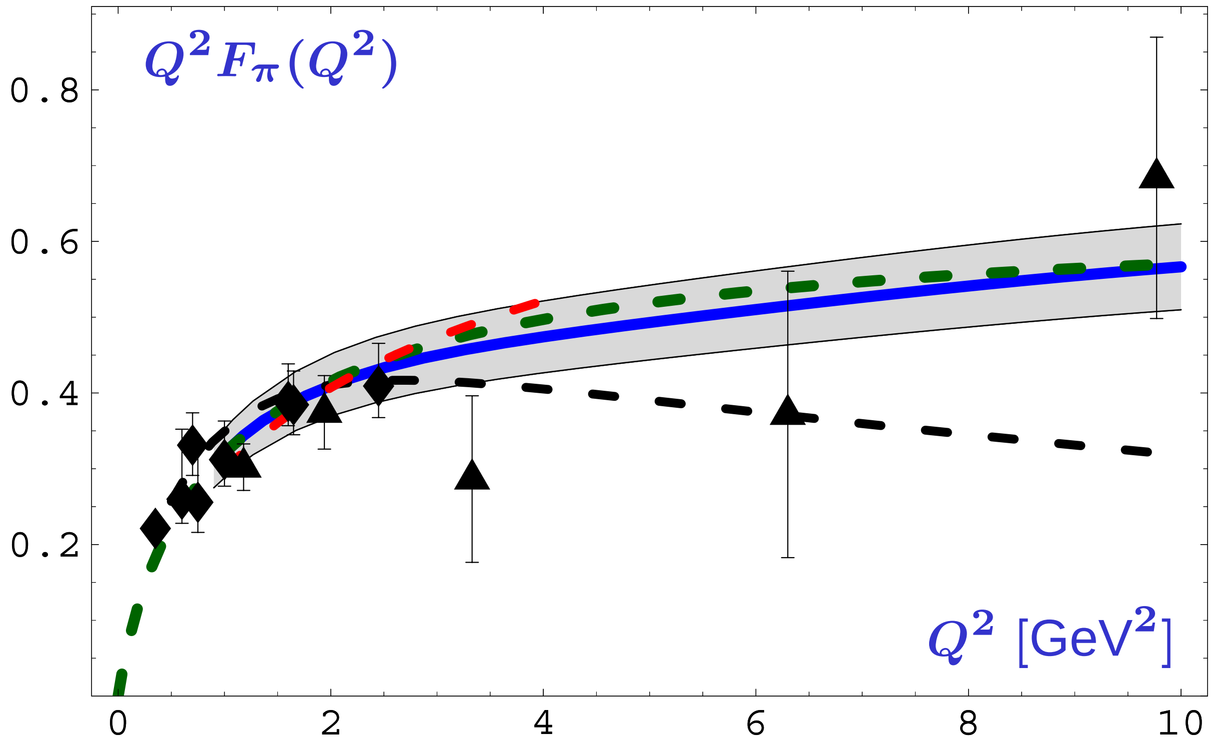

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Conclusion

- Taking into account nonlocality of condensates makes QCD SR stable and widens region of applicability up to $Q^2 \simeq 10$ **GeV^2**.
- We use the model-independent expression for Φ_{OPE}-term obtained by **Bakulev&Radyushkin**, but significantly different model of NLCs with nonlocality parameter $\lambda_q^2 = 0.4$ **GeV^2**.
- NLO corrections to double spectral density obtained by **Braguta&Onishchenko** are large to be taking into account in case of nonlocal condensate SR like in LD SR.
- QCD SR method with NLCs for the pion FF gives us a strip of predictions.
- Obtained strip for the pion FF is in a good agreement with existing experimental data.