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Метагравитация и гравискаляр

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Content

1. Metagravity

• General covariance (GC)

GC: $\delta x^\mu = \xi^\mu(x)$.

Massless (tensor) graviton g: $J = 2$, $m_g = 0$.

GC is sufficient for masslessness $J = 2$: GC $\Rightarrow m_g = 0$.

Reverse is wrong: GC is not necessary.

• Unimodular covariance (UC)

Correct statement (van der Bij, van Dam and Ng, 1982).

For masslessness $J = 2$ it is necessary and sufficient UC: $\delta x^\mu = \xi^\mu, \quad \partial \cdot \xi = 0$.

$m_g = 0 \Leftrightarrow$ UC.

Massless tensor graviton $g \oplus$ massive scalar graviton (graviscalar) h: $J = 0$, $m_h > 0$.

Most general UC metric theory $J = 2$, $m_g = 0$: "metagravity" (Pirogov, 2006, ...).

Gauge principle $\Rightarrow$ Metagravity as consistent theoretically as GR.

Phenomenologically different.

GC violation $\Leftrightarrow$ Graviscalar DM in the Universe.

• Variables

Metric field: $g_{\mu\nu}$.

UC scalar: $g \equiv \det g_{\mu\nu}$.

GC scalar density.

GC scalar: g/g_h , g_h = primordial scalar density: $g_h \sim g$.

Contracted connection:

$$\Gamma^\lambda_{\lambda\mu} = \partial_\mu \ln \sqrt{-g}.$$

Graviscalar field:

$$\chi = \frac{1}{2} \ln \frac{g}{g_h}.$$

• Lagrangian

Metagravity:

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_{gh} + \mathcal{L}_{mg} + \mathcal{L}_{mh}.$$

(i) GR.

Graviton : conventional

$$\mathcal{L}_g = -\kappa_g^2 R.$$

Matter-graviton: conventional $\mathcal{L}_{mg}$.

(ii) Beyond GR.

Graviscalar: potential: V_h

$$\mathcal{L}_h = \frac{1}{2}\kappa_h^2 \partial\chi \cdot \partial\chi + V_h(\chi).$$

Graviton-graviscalar: $\mathcal{L}_{gh} = 0$.

Matter-graviscalar: $\mathcal{L}_{mh} = 0$.

Minimal metagravity.

2. Metagravity equations

Metagravity equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{2\kappa_g^2}(T_{m\mu\nu} + T_{h\mu\nu}).$$

Matter energy-momentum tensor: $T_{m\mu\nu}$.

Graviscalar energy-momentum tensor:

$$T_{h\mu\nu} = \partial_\mu\chi\partial_\nu\chi - \frac{1}{2}\partial\chi\cdot\partial\chi g_{\mu\nu} + \check{V}_h g_{\mu\nu}.$$

Metapotential:

$$\check{V}_h = V_h + \kappa_h(\partial V_h/\partial\chi + \nabla\cdot\nabla\chi).$$

Contracted Bianchi identity:

$$\nabla_\mu(T_m^{\mu\nu} + T_h^{\mu\nu}) = 0.$$

Graviscalar $\Leftrightarrow$ DM $\oplus$ DE.

On-mass-shell condition:

$$\begin{aligned} 0 &= \nabla\cdot\nabla\chi + \partial V_h/\partial\chi, \\ R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= \frac{1}{2\kappa_g^2}\Big(T_{m\mu\nu} + \partial_\mu\chi\partial_\nu\chi - \frac{1}{2}\partial\chi\cdot\partial\chi g_{\mu\nu} + V_h g_{\mu\nu}\Big). \end{aligned}$$

Gravity GR $\oplus$ Ordinary scalar field.

Metagravity = off-mass-shell effect.

3. Spherical symmetry

• Static spherically symmetric distribution

Interval:

$$ds^2 = adt^2 - bdr^2 - cr^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

Metagravity equations:

$$\begin{aligned} R^r_r - R^0_0 &= -\frac{1}{2\kappa_g^2}\frac{1}{b}\chi'^2, \\ R^\theta_\theta - R^0_0 &= 0, \\ R^0_0 &= -\frac{1}{2\kappa_g^2}\check{V}_h. \end{aligned}$$

Metapotential:

$$\check{V}_h = V_h + \kappa_h\left(\frac{\partial V_h}{\partial\chi} - \frac{1}{\sqrt{ab}cr^2}\left(\sqrt{a/b}cr^2\chi'\right)'\right).$$

Graviscalar energy-momentum tensor:

$$T_h{}^\mu_\nu = -\frac{1}{b}\chi'^2\delta^\mu_r\delta^r_\nu + \left(\frac{1}{2b}\chi'^2 + \check{V}_h\right)\delta^\mu_\nu, \qquad \mu,\nu = 0, r, \theta, \varphi.$$

Graviscalar distribution invariant mass:

$$m_\chi = \int\left(T_h{}^0_0 - T_h{}^n_n\right)\sqrt{-g}d^3x = -2\int\check{V}_h\sqrt{-g}d^3x.$$

• Radial rescaling

Radial rescaling:

$$r \to \hat{r} = \hat{r}(r)$$

Transformations:

$$\begin{aligned} a(r) &= \hat{a}(\hat{r}(r)), \\ b(r) &= (d\hat{r}/dr)^2 \hat{b}(\hat{r}(r)), \\ c(r) &= (\hat{r}/r)^2 \hat{c}(\hat{r}(r)), \\ \chi(r) &= \hat{\chi}(\hat{r}(r)), \\ \sqrt{-g_h} &= (\hat{r}/r)^2 (d\hat{r}/dr)\sqrt{-\hat{g}_h(\hat{r}(r))}. \end{aligned}$$

Two routs:

(i) Canonical coordinates: $\hat{g}_h(\hat{r}) = -1$.

Distinguished coordinates $\Leftrightarrow$ GC violation.

Variables: $\hat{a}, \hat{b}, \hat{c}$.

Theoretical considerations. Practically unacceptable.

(ii) Observer coordinates: $F(a, b, c) = 0$.

E.g., $c = 1$ (astronomic), $c = b$ (isotropic), etc.

The third variable: χ.

$\Rightarrow g_h \Rightarrow$ Canonical coordinates: $\hat{g}_h = -1$.

• Reciprocal coordinates: $ab = 1$

Variables:

$$A = a = 1/b, \quad C = cr^2/r_0^2, \quad X = \chi/\kappa_g.$$

Free massless field: $V_h = 0$.

Metagravity equations:

$$\begin{aligned} \frac{C''}{C} - \frac{1}{2}\frac{C'^2}{C^2} &= -\frac{k_h^2}{2} X'^2, \\ \frac{A''}{A} - \frac{C''}{C} + \frac{2}{r_0^2}\frac{1}{AC} &= 0, \\ \frac{A''}{A} + \frac{A'}{A}\frac{C'}{C} &= k_h^2 \Big(X'' + \frac{(AC)'}{AC} X' \Big). \end{aligned}$$

Dimensionless parameter: $k_h = \kappa_h/\kappa_g$, $k_h \leq \mathcal{O}(1)$.

$X = 0 \Rightarrow$ GR black holes.

$X \neq 0 \Rightarrow$ "Dark holes".

4. Deformed black holes

• Exact solution

Reciprocal coordinates: $ab = 1$.

Mass-shell condition: $(ACX')' = 0$ or $ACX' = \text{Const}$.

Constrained solution:

$$\begin{aligned} A &= \Big(1 - \frac{r_\chi}{r}\Big)^{\gamma_\chi}, \\ C &= \frac{r^2}{r_0^2}\Big(1 - \frac{r_\chi}{r}\Big)^{1-\gamma_\chi}, \\ S = k_h X &= \pm\sqrt{1-\gamma_\chi^2}\,\ln\Big(1 - \frac{r_\chi}{r}\Big). \end{aligned}$$

GR $\oplus$ Ordinary scalar field (Buchdahl, 1959).

Intrinsic parameters: r_χ and γ_χ,

Consistency requirement: $\gamma_\chi^2 \leq 1$.

$\gamma_\chi = 1 \Rightarrow$ Ordinary black holes.

• Post-Newtonian approximation

Comparison with observations.

Isotropic coordinates: $\hat{c} = \hat{b}$.

Asymptotic:

$$\begin{aligned} \hat{a} &= 1 - \frac{r_g}{\hat{r}} + \frac{1}{2}\frac{r_g^2}{\hat{r}^2} + \mathcal{O}\Big(\frac{1}{\hat{r}^3}\Big), \\ \hat{b} &= 1 + \frac{r_g}{\hat{r}} + \frac{3}{8}\frac{r_g^2 - r_s^2/3}{\hat{r}^2} + \mathcal{O}\Big(\frac{1}{\hat{r}^3}\Big), \\ \hat{S} &= \mp\frac{r_s}{\hat{r}} + \mathcal{O}\Big(\frac{1}{\hat{r}^2}\Big). \end{aligned}$$

Effective parameters:

$$r_g = \gamma_\chi r_\chi, \quad r_s = \sqrt{1-\gamma_\chi^2}\, r_\chi,$$

or inversely

$$r_\chi = \sqrt{r_g^2 + r_s^2}, \quad \gamma_\chi = r_g \Big/ \sqrt{r_g^2 + r_s^2}.$$

Gravitational radius: $r_g \geq 0$.

Deviation from GR in PPN approximation. $\Rightarrow$ No observational restrictions on r_s.

5. Physics interpretation

• Metagravity

Total energy-momentum tensor:

$$T_{\mu\nu} = T_{m\mu\nu} + T_{h\mu\nu}.$$

Total mass of the deformed black hole:

$$\begin{aligned} M &= \int \left(T^0_0 - T^n_n\right)\sqrt{-g}d^3x \\ &= 4\kappa_g^2 \int R^0_0 \sqrt{-g}d^3x \\ &= 8\pi\kappa_g^2\gamma_\chi r_\chi = r_g/(2G). \end{aligned}$$

Singularity R^0_0 at $r = 0$.
Correspondence: $M \Leftrightarrow r_g$.

Graviscalar contribution at $V_h = 0$:

$$\begin{aligned} m_\chi &= \int \left(T_{h0}^{\ 0} - T_{hn}^{\ n}\right)\sqrt{-g}d^3x \\ &= -2\kappa_g \int \nabla \cdot \nabla\chi\sqrt{-g}d^3x \\ &= 8\pi\kappa_g^2 k_h\sqrt{1-\gamma_\chi^2}r_\chi = k_h\sqrt{1/\gamma_\chi^2 - 1}r_g/(2G). \end{aligned}$$

Singularity $\nabla \cdot \nabla\chi$ at $r = 0$.

Mass content:

$$M = m_m + m_\chi, \quad m_m \geq 0.$$

$\Rightarrow$ Weak parameter restriction:

$$\frac{1}{1 + 1/k_h^2} \leq \gamma_\chi^2 \leq 1.$$

In particular, $m_m = 0$ thus $\gamma_\chi = 1/\sqrt{1 + 1/k_h^2}$.
$\Rightarrow$ Spherical pure graviscalar objects ("dark balls").

• General Relativity

$V_h = 0 \quad \Rightarrow \check{V}_h = 0 \quad \Rightarrow \quad m_\chi = 0, \quad M = m_m.$

$\Rightarrow$ *Dark balls in GR are excluded theoretically .*

Matter scalar charge:

$$Q_s = -\frac{1}{\sqrt{4\pi}} \int \nabla \cdot \nabla \chi \sqrt{-g} d^3x = \sqrt{4\pi}\kappa_g \sqrt{1-\gamma_\chi^2} r_\chi = \sqrt{1/\gamma_\chi^2 - 1}\sqrt{G} M.$$

Singularity $\nabla \cdot \nabla \chi$ at $r = 0$.

Specific scalar charge:

$$q_s = Q_s/M = \sqrt{1/\gamma_\chi^2 - 1}\sqrt{G}.$$

Scalar mediated interaction:

$$\Delta V_s = -\frac{q_s^2 M m}{r} = (1 - 1/\gamma_\chi^2)\frac{GMm}{r}.$$

NL violation.

Strong parameter restriction: $q_s \to 0$ thus $\gamma_\chi \to 1$.

$\Rightarrow$ Ordinary black holes.

$\Rightarrow$ *Black holes deformed by a massless scalar field in GR are excluded phenomenologically.*

6. Conclusion

• Metagravity is consistent phenomenologically up to the post-Newtonian approximation.

• In metagravity, in contrast with GR, the scalar deformed black holes are legitimate.

• General dark hole and dark ball solutions are sensible.

• The latter solutions are mandatory to apply graviscalar as the DM resource in galaxies and the cluster of galaxies.